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Problem-Solving and Students' Use of Metacognitive Skills

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Abstract

We examined how high school students use their cognitive and metacognitive skills in the geometry problem-solving process. This research employed a mixed-methods descriptive sequential design. Data were collected in the 2019–2020 academic year at secondary education institutions in the central districts of Adana, Türkiye. Using stratified sampling, 313 students participated in the quantitative component, of 313 students 149 are girls and 164 were boys and they were all 15 years olds. Then, using extreme case sampling, 18 students were selected as participant of the qualitative component. Fourteen of them were boys and four were girls. Measures included the metacognitive skills scale, geometry problem test, thinking-aloud protocol, and an observation form. Descriptive statistics and content analysis were applied for data analysis. Results showed that students with high metacognitive skills used metacognitive skills more when solving geometry problems than students with low metacognitive skills. As the implication of the result it is suggested that attention should be paid to the development of students' metacognitive skills in schools. In this context, it would be beneficial to train teachers to develop metacognitive skills. In addition, there is a need to investigate the effect of metacognitive skills on learning in different learning areas.

Keywords: Metacognitive Skills; Problem-Solving; Geometry Problem; Secondary Education

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Introduction

Developments in the field of cognitive psychology in recent years have brought the concepts of cognition and metacognition to the forefront among learners. Especially in the process of solving mathematical problems, where cognitive processes are vital, many studies on student behaviour and the importance of metacognition stand out (Desoete and De Craene, 2019; Depaepe, De Corte, and Verschaffen, 2010; Verschaffel, Depaepe, and Mevarech, 2019; Young and Worell, 2018). In general, cognition refers to mental processes such as thinking, learning, and remembering. Metacognition includes self-regulation strategies such as awareness and self-assessment on the information learned. The concept of metacognition, first introduced by Flavell in the 1970s, is the awareness of one's own thinking processes. It is also emphasised as the ability to organise and evaluate these processes (Brown, 1987; Flavell, 1979; Scott and Levy, 2013). According to Rickey and Stacy (2000), metacognition means thinking about one's own thinking. According to Hennessey (2003), it means an individual's inner awareness about his/her learning process. For Garofalo and Lester (1985), it means knowledge and beliefs about cognitive events as well as the ability to regulate and control them. Wilson (2001) defined metacognition as an individual's awareness of his/her thinking and the ability to evaluate and organise it.

While cognitive strategies are processes used by individuals to help them reach specific goals, metacognitive strategies are the processes employed by individuals to monitor and control cognitive strategies to ensure the achievement of said goals (Livingston, 2003). Hacker (1998) defined metacognition as a person's knowledge repertoire with cognitive and affective dimensions that are constantly followed. Educators generally define thinking about thinking as the regulation of cognition (Costa and Kallick, 2000; Scott and Levy, 2013). The concept of metacognition refers to individuals' knowledge about their own information processing skills as well as the nature of cognitive tasks and strategies to cope with these tasks. It also includes managerial skills related to monitoring and individuals' self-regulation of their own cognitive activities (Schneider and Artelt, 2010).

Theoretical Framework

Types of Cognition

The concept of metacognition in education and psychology generally consists of two categories: cognition knowledge and cognition regulation (Schraw and Moshman, 1995). Cognition knowledge refers to individuals' self-control knowledge about their learning and thinking processes, which is examined within a framework of two main views stated by Flavell (1979) and Jacobs and Paris (1987). According to the theory put forward by Flavell (1979), cognition knowledge includes three dimensions: person, task, and strategy. Person knowledge means the individual's thoughts about his/her own learning, task knowledge means an individual's thoughts about a task, and strategy knowledge includes the thoughts of an individual concerning the strategic issues he/she has. According to the theory put forward by Jacobs and Paris (1987), cognition knowledge—an individual’s knowledge of a task (declarative knowledge)—includes knowing what strategies to use when performing the task (procedural knowledge) and what to do in situations related to the task (conditional knowledge; Schraw and Moshman, 1995; Verschaffel, Depaepe, Mevarech, 2019).

In contrast, strategies included in cognition regulation include three basic sub-dimensions: planning, monitoring, and evaluation. The planning stage includes the choice of strategy and the allocation of appropriate resources that affect performance. The monitoring phase refers to an individual’s instantaneous awareness of his/her understanding of the process and performance regarding the given task. The evaluation phase is the self-evaluation process about what the individual has learned and regulation (Schraw and Moshman, 1995). Many mathematics educators (e.g. Garofalo and Lester, 1985; Young and Worell, 2018) consider cognition regulation as a solution to mathematical problems. The planning stage in cognition regulation includes understanding the problem and planning, the monitoring stage includes the implementation of the solution, and the evaluation stage includes the processes of controlling the solution.

Metacognition, Problem, and Problem-Solving

Metacognition, problem, and problem-solving processes are of great importance and are the basic skills that should be acquired in mathematics education (Artzt and Armor-Thomas, 1992; Lucangeli et al., 2019; Jacobse and Harskamp, 2012; Posamentier and Krulik, 2008; Smith and Mancy, 2018). According to Blum and Niss (1991), problems are situations in which the individual does not have any methodological, procedural or algorithmic knowledge to answer, consisting of open questions and mentally challenging the individual. Problem solving, on the other hand, is the effort to reach a goal in a situation where the individual does not have an automatic solution (Schunk, 2012). Donaldson (2011) stated that problem-solving is the basis of mathematics and students should acquire these skills as they are useful in other disciplines and in their daily lives (Evans, 2012). Krulik and Rudnick (1989) emphasised the importance of using one’s own knowledge, skills, and abilities to solve problems. Jonassen (2000) stated that problem-solving is the most important tool to acquire in life.

The problem-solving process is very complex. Concerning the solving of mathematical problems, Garofalo and Lester (1985) emphasised the cognitive processes in problem-solving as preparing an action plan for a better understanding of the problem, to follow each action while implementing the plan, to select and implement appropriate strategies, to control and evaluate the results, and to review or completely change the plan when necessary. However, many mathematics educators noted that it is not enough to focus solely on cognitive processes in problem-solving (Artzt and Armor-Thomas, 1992; Schoenfeld, 1992; Smith and Mancy, 2018). In this context, many mathematics educators stated that the knowledge that students have in the problem-solving process is not sufficient in cognitive dimension. At the same time, they emphasize the importance of students' awareness that they have this knowledge and that this information should be used effectively under control (Lucangeli & Cornoldi, 1997).

Achieving the desired success in the problem-solving process means not only having field knowledge but also being aware of this information, planning and using it, organising it, controlling it, and evaluating it; in short, it also depends on metacognitive factors (Lester, 1994, Artzt and Armor-Thomas, 1992; Kramarski, Mevarech and Arami, 2002; Zhao et all., 2019; Jacobse and Harskamp, 2012; Kuzle, 2019). Metacognition helps students realise what tools they need to solve a problem and to understand how to reach a solution. Schoenfeld (1987) noted that metacognition affects success in problem-solving as well as monitoring and control of herself/himself in the process of problem-solving as much as his/her own cognitive knowledge.

In contrast, many mathematics educators working on the concept of metacognition noted that students with metacognitive skills or higher-order thinking skills are more successful at problem-solving than from students who do not have these skills (Lester, 1994; Mevarech and Kramarski, 1997; Ohtani and Hisasaka, 2018; Zhao, et al., 2019). Many mathematics educators emphasised that students who can use metacognitive skills effectively exhibit more positive attitudes towards problem-solving, which promotes success in problem-solving processes (Chan and Mansoor, 2007; Kramarski, Mevarech and Arami, 2002; Stillman and Mevarech, 2010; Smith and Mancy, 2018).

Based on some basic individual differences, Schoenfeld (1987) and Lester (1994) classified learners as high- or low-success problem solvers. In the context of metacognition, the most important of these individual differences is learners' awareness of knowledge, ability to control weaknesses and strengths, and ability to organise and monitor the problem-solving process and reach conclusions (Erbas and Okur; 2012; Young and Worell, 2018). In addition, high-success problem solvers deal with the structural properties of problems, while low-success problem solvers focus on problems without paying attention to the details (Schoenfeld, 1987). Prior studies concerning mathematics education indicated that there is a positive difference in the metacognitive skills and problem-solving skills of primary and secondary school students who can organise metacognitive processes and information on their own and use learning strategies appropriately and correctly (Schneider and Artelt, 2010; Desoete et al., 2001; Kuzle, 2018, 2019; Smith and Mancy, 2018; Veenman and Cleef, 2019).

In this study, the achievement levels of students with low and high metacognitive skills, who were asked to solve geometry problems, were examined. We chose geometry because, children in many countries score the lowest on geometry in Programme for International Student Assessment (PISA) exam (Sorby and Panther, 2020). Another important reason for choosing the geometry subfield is that prior studies that examined metacognitive and problem-solving skills typically analysed algebra (Bani, Ekawati, Rahaju, 2019; Desoete et al., 2001; Erbas and Okur, 2012; Jacobse and Harskamp, 2012; Özkubat and Özmen, 2021; Teong, 2003). Scant studies have addressed metacognitive skills and geometry (Susanto and Irvan, 2018; Kuzle, 2017; Rofii, Sunardi and Irvan, 2018). Moreover, although many researchers have examined metacognition in younger age groups, it remains unclear how this process occurs in adolescence (Stillman and Mevarech, 2010). This is valuable since this age group can participate in international exams such as PISA and the Trends in International Mathematics and Science Study.

Consequently, we examined how high school students use their cognitive and metacognitive skills in the geometry problem-solving process and aimed to answer two research questions:

1) What are students' metacognitive skill levels?

2) How do students with low and high metacognitive skills use cognitive and metacognitive skills in the problem-solving processes related to geometry?

Methodology

This study followed a mixed-methods sequential explanatory design. First, quantitative data were collected and analysed; second, qualitative data were collected and analysed; then, the results were integrated (Creswell and Poth, 2017; Shorten and Smith, 2017). As quantitative data, the scores obtained from the metacognitive skill scale were used to determine the metacognitive skill levels of the students. Later, 18 students who got low and high scores from this scale were determined and qualitative data were collected from these students.

Population and Sample

Concerning the quantitative component, using stratified sampling, we recruited students in the 2019–2020 academic year from secondary education institutions in the central districts of Adana (Cukurova and Sarıcam), Türkiye. Stratified sampling enables the determination of sub-groups in the population and their representation in the population (Mills and Gay, 2019). In contrast, the sub-groups of this study were the schools that accept students with a certain exam score and the schools that accept students regardless of their exam score. The metacognitive skills scale was applied to 191 students from the schools attended by students who scored high in the high school entrance exam and 122 students from 122 students from the schools attended by students who scored low in the high school entrance exam. Of 313 students 149 are girls (48%) and 164 are boys (52%). Further, 25% (n = 78) scored 0–49 grade points, 5% (n = 16) scored 50–59 grade points, 6% (n = 19) score 60–69 grade points, 28% (n=88) scored 70–84 grade points, and 36% (n = 112) scored 85–100 grade points. All students are 15 years old.

Eighteen students formed the qualitative sample. These students were recruited using purposeful sampling according to extreme case sample, one in which all of the members are outliers who do not fit the general pattern or who otherwise display extreme characteristics (Fraenkel, Wallen, 2018). Extreme cases consist of students who score low and high on the metacognitive awareness scale. Interviews were conducted in the school's administrative office or in a multi-purpose hall from February–March 2020. The duration was not restricted; but interviews lasted between 45 minutes and 60 minutes.

The qualitative sample was determined by analysing students' scores on the metacognitive skills scale: nine students with the lowest scores $\overline{(\mathrm{X}} = 1{,}67 - 2{,}1)$ and nine students with the highest scores $\overline{(\mathrm{X}} = 4{,}50 - 4{,}94)$. The students in the low metacognitive skills group scored between 5 and 64 (seven boys and two girls). All of these students were from the schools attended by students who scored low in the high school entrance exam. The students in the high metacognitive skills group scored between 78 and 100 (seven boys and two girls). Seven of these students attended schools accepting student high scores in the high school entrance exam, while the other two were from school accepting low scores in the high school entrance exam. Coding was used to protect students' anonymity: low = L, and high = H, and numbers were used for interview order; thus, L1 was the first interviewed student with low metacognitive skills, H1 was the first interviewed student with high metacognitive skills, and so on.

Data Collection Tools

We utilised the metacognitive skills scale, the geometry problem test, the thinking-aloud protocol, and an observation form to collect data. Both concurrent (Ader, 2019; Shilo and Kramarski, 2019) and asynchronous (Lucangeli et al., 2019; Zhao et al., 2019) techniques were used to evaluate cognitive and metacognitive strategies. To benefit from the strengths of both techniques and to be less affected by their limitations, simultaneous and non-simultaneous techniques should be used together (Veenman and Cleef, 2019). Specifically, the metacognitive skills scale is an asynchronous example, and the think-aloud protocol is a synchronous example.

Metacognitive Skills Scale

The Junior Metacognitive Awareness Inventory-B, developed by Sperling et al. (2002), was used to determine students' perception of their own learning structure and awareness of their learning characteristics. The scale comprises 17 items that are answered using a 5-point Likert scale: 5.00–4.20 = 'always', 4.19–3.40 = 'often', 3.39–2.60 = 'sometimes', 2.59–1.80 = 'rarely', and 1.79–1.00 = 'never'.

An item sample for each sub dimension is presented below.

I ask myself questions about how well I am learning while I am learning something new (cognition knowledge)

I ask myself periodically if I am meeting my goals (cognition regulation)

In a previous exploratory factor analysis of the validity and reliability of the Turkish version, Aydın and Ubuz (2010) found that the scale consists of two dimensions: cognition knowledge and cognition regulation. Cronbach's Alphas were 0.74 and 0.79, respectively.

Geometry Problem Test

A geometry problem test was developed by the current researchers. It consists of six problems related to learning geometry. During the preparation process, the relevant literature was examined and problems that would reveal students' metacognitive processes were created. The questions were open-ended, paralleled the school's geometry curriculum, and were related to students' daily lives, in accordance with their grade level (e.g. Mevarech and Kramarski, 1997). In addition, the problems required a solution that can reveal the planning, monitoring, and evaluation steps from the metacognitive skills revealed by Shraw and Monter (2005). To ensure test validity, an expert panel (three mathematics teachers and one curriculum developer) was formed. Owing to their feedback, two questions were removed and one question was modified. When the difficulty levels of the problems were examined, the first two questions were about measuring length and were deemed easy to solve; the next two questions were about area calculation of geometric figures and were moderately difficult; and the last two questions were related to area and volume, which could be solved in three steps and were deemed difficult (Rosenzweig, Krawec, and Montague, 2011). We wanted to include varying difficultly levels to determine whether difficultly was associated with the strategies that students used in problem-solving processes.

One of the problems asked is presented below.

Mr. Ali wants to feed chickens in his farm. For this, he wants to create a poultry house area with a 24-meter wire with the back wall of his house as the border. For the chickens to move freely in the coop to be set up by Mr. Ali, how many square meters can the area of this coop be at most?

Think-Aloud Protocol

The think-aloud protocol aims to reveal the internal thoughts or cognitive processes of the participants about existing situations or phenomena during task performance (Patton, 2015). In other words, the think-aloud protocol is an important multidimensional assessment technique that is used to measure participants' verbal performance that is in fact being assessed their internal experiences, and what strategies they employed (Rosenzweig et al., 2011). It is one of the most appropriate methods used to examine cognitive processes such as problem-solving, especially in mathematics (Jacobse and Harskamp, 2012; Veenman and Cleef, 2019). Students were encouraged by the researchers to verbally express their thoughts during the problem-solving process. Students' responses were audio-recorded.

Observation Form

A structured observation form was developed by the current researchers. The form was created by conducting informal interviews with students, observing students' problem-solving behaviours, and reviewing the relevant literature (Artzt and Armor-Thomas, 1992; Garofalo and Lester, 1985; Montague, Applegate, 1993; Kuzle, 2017). The draft observation form was prepared by considering two behavioural classifications that students display in problem-solving processes: cognitive and metacognitive. After the form was reviewed by three mathematics educators, it was finalised. The cognitive process comprised reading, understanding, expressing the problem in their own words, visualising, planning, predicting, processing, and evaluating. The metacognitive process comprised planning (self-instruction), monitoring (self-questioning), and self-evaluation (self-monitoring).

Data Analysis

Data were analysed in accordance with the mixed-methods descriptive sequential design; i.e. quantitative data were analysed before qualitative data (Creswell, 2013). Descriptive statistics were applied to the quantitative data obtained from the metacognitive skills scale, which were used to determine the qualitative sample (as noted above). Qualitative data were analysed by using SPSS version 22 (SPSS; IBM, Armonk, NY, USA). Qualitative data were content analysed.

First, the data obtained from the think-aloud protocol were recorded by the researchers and turned into a written document. Second, the written document was coded by the researchers using the problem-solving steps. Third, raw data obtained from the voice recordings of five students were randomly analysed by two experts in the field of mathematics education, who encoded the data. The consensus between coders was 92%, indicating strong reliability (i.e. it should exceed 70%; Miles, Huberman, and Saldana, 2014). Differences between coders were resolved by modifying or removing the code until consensus was reached. After the coding of the think-aloud protocol, the researchers combined the data obtained as a result of the observation, which could not be detected in the think aloud protocol, with the coding, taking into account the literature on the codings (Artz and Armor-Thomas, 1992; Rosenzweig et al., 2011; Teong, 2003).

Validity and Reliability

In the current study for the Metacognitive Skills Scale, an exploratory factor analysis revealed a two-factor structure that explained 46.7% of the total variance, and an eigenvalue above 1 was obtained. According to the confirmatory factor analysis results (χ^2/df = 191.14/118) = 1.61, $p < .001$;

Goodness of Fit Index (GFI) = 0.93, Incremental Fit Index (IFI) = 0.98, Comparative Fit Index (CFI) = 0.98, Adjustment Goodness of Fit Index (AGFI) = 0.91, Root Mean Square Error of Approximation (RMSEA) = 0.046), the model had an acceptable goodness-of-fit (Jöreskog and Sörbom, 1993). Cronbach's Alphas were 0.85 for cognition knowledge, 0.83 for cognition regulation factor, and 0.91 for the total scale.

To ensure the validity and reliability of the qualitative data, rich descriptions, diversification, and expert control strategies were used. In this context, the qualitative data obtained were presented as codes and themes for the purpose of rich description and supported with direct quotations. In order to increase the reliability and validity of the data, the thinking aloud protocol and the data obtained from the observation were cross-checked. In addition, a second encoder was used in the analysis of qualitative data. The agreement between the coders was calculated as .92 according to the formula of Miles and Huberman (2015).

Ethical Consideration

Written permission was obtained from the Directorate of National Education and from the schools within the scope of the research (Permission report date: 05.12.2019, number: E.46678). In addition, written consent was obtained from the parents for their children to participate in the study. The students participating in the study were asked verbally whether they were volunteers or not, and it was stated that non-volunteers could not participate in the study or leave the research process. Additionally, an ethical approval certificate was obtained from the scientific ethics committee of Cukurova University

Findings

Students' scores on the metacognitive skills scale are presented in Table 1: cognition knowledge ($\overline{X}$ = 3.71), cognition regulation ($\overline{X}$ = 3.97), and total score ($\overline{X}$ = 3.84). Thus, students frequently used both cognition knowledge and cognition regulation.

Table 1. Students' scores on the metacognitive skills scale.

Factor	N	$\overline{X}$	SD	Lower and upper limit
Cognition knowledge	313	3.71	.67	2.11-4.89
Cognition regulation	313	3.97	.60	1.22-4.89
Total	313	3.84	.59	1.67-4.94

The themes and codes for the cognitive skills used by students with low or high metacognitive skills in the problem-solving process are shown in Table 2. Four key categories emerged: *understanding the problem*, *planning*, *solving the problem*, and *evaluating the problem*.

Table 2. Cognitive strategies used by students with low and high cognitive skills in the problem-solving process.

Category	Code	Low metacognitive skills (n = 9)							High metacognitive skills (n = 9)						
	CB	E21	E22	M 1	M 2	D 1	D 2	T	E 21	E 22	M1	M 2	D1	D2	T
		f	f	f	f	f	f	f	f	f	f	f	f	f	f
Understanding the problem	Read	9	12	11	13	16	12	73	11	12	14	15	12	14	78
Planning	Implementing the plan	8	3	7	5	4	7	34	8	5	3	4	10	12	42
	Trial-and-error strategy					3		3		2					2
Problem-solving	Calculation	6	13	6	9	6	7	47	5	5	7	3	14	11	45
Evaluating the problem	Evaluating the result	1	1	4	2	1	6	15	3	2	2	4	1	7	19
	Total	24	29	28	29	27	32	169	27	24	26	26	37	44	184

CB: Cognitive Behaviour; E: Easy; M: Moderate; D: Difficult; T: Total

First, *understanding the problem* was used by students with both low (f = 73) and high (f = 78) cognitive skills. Second, *planning* was frequently used by students with both low (f = 34) and high (f = 42) cognitive skills. When the difficulty levels of the questions were examined, students with low and high cognitive skills made plans with similar frequency for easy and moderately difficult questions; however, students with high cognitive skills made more plans for difficult questions than did those with low cognitive skills. For example, H1 noted, '*How many pieces are there now? I'll find this first. So, if he bought 24 pieces in 50 minutes, then how many minutes would he get a piece? I have to find 50/24. It should fall on each piece in 25/12 minutes. I have to calculate it*'. Moreover, H2 said, '*First I will find the height of the cabinet from the ground and divide it into two*'. In contrast, students did not use the trial-and-error strategy much in the planning category. For example, L9 said, '*...I can give a number in the head. The number I will give is divided by all of them. Okay, if there were 12 coming here, they would all be divided by 12*'. Additionally, L5 said, '*The green ceramic box is at least that. Now I can think of a common multiple of all of them; but, if I do so, I think it will be too long. That's why I say I think it should be blue*'.

Third, *problem-solving* was frequently used by students with both low (f = 47) and high (f = 45) cognitive skills. For example, H1 said, '*If there is a 24-metre wire, now I can extend the wall as much as I want. Let the limit be 1. If one side is 1 metre, the other side is 1. Then its other side is 24-(1+1) = 22 and half is 22:2 = 11. Then the area of the coop would be 1 × 11 = 22 metres*. In the form, he calculated the result of the problem. Similarly, H4 said, '*...He was 0.3 metres, or 30 centimetres, from the wall. I found the area of 150 square metres. So, when I divide 150 by 30, I find that there should be 150:30 = 5 tables.*

It was observed that students with low (f = 19) and high (f = 15) cognitive skills not often checked the results of problem solving. In addition, these two groups of students demonstrated this behavior at close frequency to each other. For example, H2 said, '…*While checking the solution of this problem, he first gave the starting point in the question and asked where he could be after half an hour. I wonder where half a turn will pass. First, I thought, when it passes half a turn, it will likely be somewhere between C and D*'.

The categories and codes for the metacognitive skills used by students with low and high metacognitive skills in the problem-solving process are shown in Table 3.

Table 3. Metacognitive strategies used by students with low and high metacognitive skills in the

Category	Code	Low metacognitive skills (n = 9)							High metacognitive skills (n = 9)						
		E21	E 22	M 1	M2	D 1	D2	T	E21	E 22	M 1	M 2	D 1	D 2	Total
		f	f	f	f	f	f	f	f	f	f	f	ff	f	f
Understanding the problem	Expressing in your own words	4	4	10	7	10	7	42	7	8	17	21	24	13	90
	Checking the information given	8	3	7	5	4	7	34	8	10	3	4	10	12	47
	Visualisation	0	1	4	2	1	6	14	3	2	2	2	1	7	17
	Total	12	8	21	14	15	20	90	18	20	22	27	35	32	154
Planning	Before starting to solve the problem, explain the path followed and visualise and interpret it in one's mind	1	2	7	7	3	9	30	4	1	7	5	18	18	53
	Facilitating operations				1			1	0	3					3
	Total	1	2	7	8	3	9	31	4	4	7	5	18	18	56
Implementation (monitoring)	Self-instruction	1					1	1	8	8	11	10	13	13	63
	Explain the reasons for his problem solution			1			1	2	0	4	1		1		6
	Be aware if something is wrong	1		1			2	2	2	1	5	2	5	5	20
	Total	2		2			4	4	10	13	17	12	19	18	89
Evaluation	Asking oneself questions by thinking about the steps to solve the problem				1			1	7	8	5	9	4	8	41
	Using another strategy to solve the problem								8	1	0	1	2	1	13
	Total				1			1	15	9	5	10	6	9	54
Overall total		13	10	30	23	15	33	125	47	46	51	54	78	77	353

E: Easy; M: Moderate; D: Difficult; T: Total

Four key categories emerged: *understanding the problem, planning, implementation (monitoring),* and *evaluation.* First, *understanding the problem* was used more by students with high cognitive skills (f = 90) than students with low cognitive skills (f = 42). Concerning difficulty, students in both groups expressed more problems in their own words in the moderately difficult and difficult questions than the easy questions. H1 said, '*Okay, they want to create a poultry house. Mr. Ali asks how many square metres can the area be for the chickens in the coop to move freely*'. In addition, students with high cognitive skills (f = 17) and students with low cognitive skills (f = 14) rarely used the visualisation code (Drawing shapes, comparing shape and problem information). For example, L8 said '*...My house will stand straight and then there will be chickens. It will be easy for chickens to enter here from outside. I can draw the house over there. Let this be the farm. There are chickens; so, the back wall of his house is intertwined with the farm. So, let's call it home. Wall length can be extended as desired. I can create an area in such a 24 metre shape by using this wall…*'

Second, compared to students with high cognitive skills (f = 53), students with low cognitive skills (f = 30) used *planning* more frequently. In addition, when the difficulty levels of the questions are examined, it can be said that high-level students plan more. For example, H5 said, '*Okay, then I will do this. First, I delete one of the squares in the places where there is a wall; I draw. Because he will not be able to come here. Then I choose any corner for the table sets. I start placing the table sets there. I start from there, respectively. I place them with a space between them and then count them one by one. So, I put a table set in every 1 corner. I guess there is probably a table in the middle of the room. Let's calculate now*'. Additionally, L1 said, '*He wants the number of teams to be placed in the sitting area in the shop. In other words, I think to do it not by trading, but by shape. First, it should be away from the walls and I have to mark each of them as a square space*'. Within the same category, it was clearly seen that the code facilitating operations were rarely used students with high (f = 3) and low (f = 1) cognitive skills. For example, L6 said, '*If there were no decimal numbers 0,3. If this was 3, then I would multiply this place by 10 and it would be 3; so, I think it would cover 3 squares. Ok, it will cover 3 squares*'. Similarly, H5 said, *'If each one is 0.3 metres, I can't deal with the decimal fraction; let's say 0.3 = 3/10. Now let me convert that to centimetres. Here, 1 metre is 100 cm. Then, 100 × 3/10 times = 30 cm*'. He converted the decimal number to a fraction and facilitated the process by using the relationship between metres and centimetres.

Third, the *implementation* category comprised self-instruction, explaining one's work, and being aware of one's mistakes. As in other categories related to metacognition, students with high (vs. low) cognitive skills mostly used this category and most utilised self-instruction (f = 63). As the difficulty level of the questions increased, the frequency of using this strategy also increased. For example, H1 said, '*Its total diameter is 230. Yes, the highest point he wants from us; it was wanted how many metres above Seyhan Lake. The farthest point A is 230 metres. First, I found half of it.*

230:2 = 115 okay. There were 20 here too. 115 plus 20 is 135'. In contrast, the least used code in this category was explaining the reasons for their actions. While six students with high cognitive skills did this, only two students with low cognitive skills did. L1 said, '*Now this is the wall. So, I put only one table set here; then, I count each square one by one, and there are 10 table sets in total. But I probably made a mistake in the account. Let me check. I put 1, 2, 3, 4, 5, 6, 7, 8. I counted wrong: 8 tables*'. One of the least-used strategies was being aware of one's mistake. H2, who used this code, said, '*He wants a coop area with the back wall of the house as the border. But this area should be a place where chickens can roam free. I can set that back wall whatever I want. Then the other three sides will be equal to each other. What could be the area of the coop: 1 × 24 = 2 × 12 = 3 × 8 = 4 × 6 = 24. If it is 8 × 3, which one will have a larger area? I have to value. All of them will give the truth. Why did I do that so that all of their fields will give 24. But, sorry, I had to find the length of the three sides. I made [a] mistake. In this question, since 3 sides are 24, I had to divide 24 by 3, and I was going to find an edge and calculate its area from there*'.

Lastly, the *evaluation* category comprised asking oneself questions by considering the steps to solve the problem and using other strategies to solve the problem. Most codes were used by students with high cognitive skills, and asking oneself questions (f = 41) and considering the steps to solve the problem were used most frequently. In addition, it can be said that students with low and high metacognitive skills use this strategy closely according to the difficulty levels of the questions. For example, H4 said, '*I found the area 150 square metres. So, when I divide 150 by 30, I find that there should be 150:30 = 5 tables. Is it true? Let me see. Now there were 5 tables. If each one is 30 cm, then 30 × 5 = 150 is ok. Yes, it would be right, okay there will be 5*'. In contrast, using other strategies was the least used; On the other hand, it is observed that the use of other strategies is applied very rarely to solve the problem. It is understood that the use of other strategies is mostly used in the process of evaluating the solution of easy problems. For example, H1 said, '*He/she can complete his/her circle in 50 minutes. For this, I had to find the environment first. But since the value of was not given in the question, I also applied the ratio-ratio. In the question, if there is a tour going every 50 minutes, I have established a proportion of how much it goes in half an hour*'.

Discussion

This research was conducted to examine the cognitive and metacognitive skills used by high school students when solving geometry problems. Most students used the cognitive knowledge and cognition regulation components of the metacognitive skills scale 'frequently'. In other words, students had high metacognitive skills, which was similar to the relevant literature (Aljaberi and Gheith, 2015; Aydın and Ubuz, 2010; Young and Worell, 2018). For example, Young and Worell (2018) concluded that middle school and high school students' metacognitive skills for problem-solving are quite high.

In contrast, when the cognitive skills used by students in the problem-solving process were examined according to our second sub-purpose, students with high cognitive skills used these skills more than students with low cognitive skills. This result is similar to the studies in the related literature (Erdinç and Okur, 2012; Özkubat and Özmen, 2021). For example, Özkubat and Özmen (2021) found that low achieving students used cognitive strategies less than average students when solving mathematical problems.

We concluded that students with high metacognitive skills used metacognitive skills more when solving geometry problems than students with low metacognitive skills, which mirrors the related literature (Artz and Armor-Thomas, 1992; Montague and Applegate, 1993; Desoete et al., 2001; Rosenzweig et al., 2011; Serin and Korkmaz, 2018; Schoenfeld, 1987). For example, Serin and Korkmaz (2018) found that students who are more successful at mathematical problem-solving use metacognitive process more so than other students. Rosenzweig et al. (2011) revealed that students with moderate achievement use higher cognitive skills more in problem-solving than their low achievement peers. Similarly, Deosete et al. (2001) found that students with high mathematical problem-solving skills have more metacognitive knowledge and skills than students with low mathematical problem-solving skills. In contrast, Swanson (1990) found that students with high metacognitive skills used metacognitive processes such as planning (hypothetical deduction (if-so propositions) and evaluation) more effectively than students with low metacognitive skills.

We also concluded that although all the students read the problems in a similar way, it was observed that students with high metacognitive skills expressed the problem more by using their own sentences, whereas students with low metacognitive skills used their own sentences less in the expression of the problem. This result is also similar to what has been shown earlier (Aydemir and Kubanc, 2014; Schoenfeld, 1987; Veenman and Cleef, 2019). Veenman and Cleef (2019) revealed that students with high metacognitive skills read problems in a way that spurs prior knowledge and allows them to determine what is required in the problem-solving process. Similarly, Sutherland (2002) stated that all students read the text given in the problem; but students who had difficulty solving the problem could not understand what was required, and they were unable to comment on the problem.

All students explained (planned) what operations they would apply when problem-solving; however, students with high metacognitive skills explained their planning and what strategy they used more so than their counterparts. This coincides with prior literature (Aydemir and Kubanc, 2014; Cozza and Oreshkina, 2013; Schoenfeld, 1987; Veenman and Cleef, 2019). For example, Aydemir and Kubanc (2014) concluded that students who use metacognitive skills can explain why they chose what strategy they planned to use; however, students with low metacognitive skills chose a strategy at random during the planning stage. Further, Montague (1992) revealed that low-level problem solvers spend a lot of time on operations and formulas rather than making plans while problem-solving. In addition, students who were successful in the problem-solving process employed effective strategies such as relating and critical processing information, self-regulation of attention, effort, and task persistence when they are faced with a problem (Veenman and Cleef, 2019).

All students used calculation strategies in the problem-solving process; however, students with high metacognitive skills mostly used the strategies of controlling their own instructions and evaluating their plans in this process. This too coincides with previous literature (Artz and Armor-Thomas, 1992; Cozza and Oreshkina, 2013; Depaepe, De Corte, and Verschaffen, 2010; Montague, 2001; Özkubat and Özmen, 2021; Veenman, 2017). Montague (2001) found that successful problem-solving students could organise themselves, give instructions, question themselves, and observe themselves during the implementation phase of problem-solving. Özkubat and Özmen (2021) also revealed that middle school students mostly used the calculation strategy when solving mathematical problems, which may be because this strategy is emphasised in schools.

Finally, although few students could control the outcome of the problem operationally, students with high metacognitive skills asked themselves questions by thinking about the steps needed to solve the problem and evaluated the effectiveness of the strategies they applied. These results also coincide with past studies (Cozza and Oreshkina, 2013; Erbas and Okur, 2012; Purnomo, Toto, Subanji, and Swasono, 2016; Veenman and Cleef, 2019). Erbas and Okur (2018) found that students who were both successful and unsuccessful in solving verbal mathematical problems not utilised what they had learned from the previous problem when solving the next problem. Similarly, Aydemir and Kubanc (2014) revealed that students with high metacognitive skills checked the accuracy of their answers when problem-solving and questioned the process. Purnomo et al. (2016) revealed that students with high metacognitive skills think of different solutions that they can use when problem-solving.

In sum, we concluded that students with high metacognitive skills use metacognitive skills more when solving geometry problems than students with low metacognitive skills. We also examined the sub dimensions of cognitive skills and how they are associated with students' ability to solve geometry problems. Training can be provided to facilitate and improve students with different cognitive skills to use metacognitive strategies while solving geometry problems. Research results revealed the importance of metacognitive skills in solving geometry problems. For this reason, attention should be paid to the development of students' metacognitive skills in schools. In this context, it would be beneficial to train teachers to develop metacognitive skills. Although the research results were obtained from 18 students with a qualitative method, similar results were obtained with the studies on the same subject in the literature. Therefore, the finding also is thought to be applicable to other countries for similar age groups.

In addition, this study shed light on how students understand math problems, how they analyse the problems, how they develop solutions, how they complete relevant tasks, and how they evaluate the results. These have widespread implications for mathematics educators, who can tailor the difficulty of the geometry problems to fit students' skillsets and promote higher cognitive skills and metacognition.

Limitations

This study had some limitations. First, the use of the think-aloud protocol as a data collection tool assumes that students can think aloud while completing tasks. However, students may also have non-verbal metacognitive skills that could not be ascertained. Further studies should employ larger sample sizes and students with varying abilities (e.g. gifted students or students with learning difficulties). Lastly, the current sample comprised ninth-grade students; therefore, future studies should examine the problem-solving processes of students across different grade levels. In addition, there is a need to investigate the effect of metacognitive skills on learning in different learning areas.

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References

Ader, E. 2019. "What Would You Demand Beyond Mathematics? Teachers' Promotion of Students' Self-Regulated Learning and Metacognition." *ZDM Mathematics Education 51* (4): 613–624. doi:10.1007/s11858-019-01054-8.

Aljaberi, N. M., and E. Gheith. 2015. "University Students' Level of Metacognitive Thinking and Their Ability to Solve Problems." *American International Journal of Contemporary Research* 5 (3): 121-134. http://www.aijcrnet.com/

Artzt, A. F., and E. Armour Thomas. 1992. "Development of a Cognitive-Metacognitive Framework for Protocol Analysis Of Mathematical Problem Solving in Small Groups." *Cognition and Instruction* 9 (2): 137–175. doi:10.1207/s1532690xci0902_3

Aydemir, H., and Y. Kubanc 2014. "Investigation of the Cognitive Behavioral Problem Solving Process." *Turkish Studies* 9 (2): 203-219. doi:10.7827/TurkishStudies.6555

Aydın, U., and B. Ubuz 2010. "Turkish Version of The Junior Metacognitive Awareness Inventory: the Validation Study." *Education and Science 35:* 30–42.

Bani, B. S., E. Ekawati, and E. B. Rahaju 2019. "Metacognition Behavior of Junior High School Students in Solving Algebra Problems in Terms of Mathematical Abilities." *International Journal of Scientific and Research Publications* 9(9): doi:10.29322/IJSRP.9.09.2019.p9310

Blum, W., M. Niss,1991. "Applied mathematical problem solving, modelling, applications, and links to other subjects – state, trends and issues in mathematics instruction." *Educational Studies in Mathematics* 22 (1): 37-68. doi:10.1007/BF00302716.pdf

Brown, A. 1987. "Metacognition, Executive Control, Self-Regulation and Other More Mysterious Mechanisms." In *Metacognition, motivation and understanding*, edited by F. E. Weinert and R. H. Kluwe, 65–116. Hillsdale, NJ: Lawrence Erlbaum Associates.

Costa, L.A., and B. Kallick. 2000. "Getting into the Habit of Reflection." *Educational Leadership,* 57 (7): 60-62.

Cozza, B., and M. Oreshkina 2013. "Cross-cultural Study of Cognitive and Metacognitive Processes During Math Problem Solving. *School Science and Mathematics*, *113*(3), 275-284.

Creswell, J. W. 2013. *Qualitative Inquiry and Research Design*. London, UK: Sage.

Creswell, J. W., and C. N. Poth, C. N. 2017. *Qualitative Inquiry and Research Design: Choosing Among Five Approaches.* London, UK: SAGE Publications, Inc.

Depaepe, F., E. De Corte, and L. Verschaffel. 2010. "Teachers' Metacognitive and Heuristic Approaches to Word Problem Solving: Analysis and Impact on Students' Beliefs and Performance." *ZDM Mathematics Education* 42: 205-218. doi:10.1007/s11858-009-0221-5.

Desoete, A., and B. De Craene. 2019. "Metacognition and Mathematics Education: An Overview." *ZDM* 51: 565–575. https://doi.org/10.1007/s11858-019-01060-w

Desoete, A., H. Roeyers, and A. Buysse. 2001. "Metacognition and Mathematical Problem Solving in Grade 3." *Journal of Learning Disabilities*, 34 (5): 435-449.

Donaldson, S. E. 2011. "Teaching through problem solving: practices of four high school mathematics teachers." PhD diss., University of Georgia.

Erbas, A. K., and S. Okur. 2012. "Researching Students' Strategies, Episodes, And Metacognitions in Mathematical Problem Solving." *Qual Quant,* 46: 89–102. doi:10.1007/s11135-010-9329-5

Evans, N. 2012. "Destroying Collaboration and Knowledge Sharing in The Workplace: A Reverse Brainstorming Approach." *Knowledge Management Research and Practice,* 10: 175–187. doi:10.1057/kmrp.2011.43

Flavell, J. H. 1979. "Metacognitive and Cognitive Monitoring." *American Psychologist,* 34: 906- 911. doi:10.1037/0003-066X.34.10.906

Fraenkel, J. R., N. E. Wallen, and H. H. Hyun. 2012. *How to Design and Evaluate Research in Education.* New York: McGram-Hill Companies.

Garofalo, J., and F. K. Lester. 1985. "Metacognition, cognitive monitoring, and mathematical performance." *Journal for Research in Mathematics Education,* 16: 163-176. doi:10.5951/jresematheduc.16.3.0163.

Hacker, D. J. 1998. "Definitions and Emprical Foundations." In D. *The Educational Psychology Series. Metacognition İn Educational Theory And Practice, edited by* J. Hacker, J. Dunlosky, and A. C. Graesser, 1-25. New York: Routledge.

Hennessey, B. A. 2003. "The Social Psychology of Creativity." *Scandinavian Journal of Educational Research,* 47 (3): 253-271. doi:10.1080/00313830308601

Jacobs, J., and S. Paris. 1987. "Children's Metacognition about Reading. Issues in Definition, Measurement and Instruction." *Educational Psychologist,* 22: 255-278. doi:10.1207/s15326985ep2203&4_4

Jacobse, A. E., and E. G. Harskamp 2012. "Towards efficient measurement of metacognition in mathematical problem solving." *Metacognition and Learning* 7 (2): 133-149. doi:10.1007/s11409-012-9088-x

Jonassen, D. H. 2000. "Toward a Design Theory of Problem Solving." *Educational Technology Research and Development* 48: 63–85.

Jöreskog, K. G., and D. Sörbom. 1993. *Lisrel 8: Structural equation modelling with the SIMPLES common language.* Chicago: Scientific Software International.

Kramarski B., Z. R. Mevarech, and M Arami 2002. "The Effects of Metacognitive Training on Solving Mathematical Authentic Tasks." *Educational Studies in Mathematics* 49, 225-250. doi:10.1023/A:1016282811724

Krulik, S., and J. A. Rudnick. 1989. *Problem solving: A handbook for senior high school teachers.* Needham Heights, MA: Allyn & Bacon.

Kuzle, A. 2017. "Delving into the Nature of Problem Solving Processes in a Dynamic Geometry Environment: Different Technological Effects on Cognitive Processing." *Technology, Knowledge and Learning* 22 (1): 37–64. doi:10.1007/s10758-016-9284-x

Kuzle, A. 2018. “Assessing Metacognition of Grade 2 and Grade 4 Students Using an Adaptation of Multi-Method Interview Approach During Problem Solving.” *Mathematics Education Research Journal* 30 (2): 185–207. doi:10.1007/s13394-017-0227-1

Kuzle, A. 2019. “Second Graders’ Metacognitive Actions in Problem Solving Revealed Through Action Cards.” *The Mathematics Educator* 28 (1): 27–60. https://eric.ed.gov/?id=EJ1225418

Lester, F. K. 1994. “Musing about Mathematical Problem Solving Research: 1970-1994.”, *Journal for Research in Mathematics Education* 25 (6): 660-675. doi: 10.2307/749578

Livingston, J. A. 2003. “Metacognition: An Overview.” ERIC Document Reproduction Service No. ED474273.https://files.eric.ed.gov/fulltext/ED474273.pdf.

Lucangeli, D., and C. Cornoldi. 1997. “Mathematics and Metacognition: What is the Nature of the Relationship?” *Mathematical Cognition* 3: 121–139. doi: 10.1080/135467997387443

Lucangeli, D., M. C. Fastame, M. Pedron, A. Porru, V. Duca, P. K. Hitchcott, and M. P. Penna. 2019. “Metacognition and Errors: The Impact of Self-regulatory Trainings in Children with Specific Learning Disabilities.” *ZDM* 51 (4): 577–585 doi: 10.1007/s11858-019-01044-w.

Merriam, S. B., and E. J. Tisdell. 2013. *Qualitative Research: A Guide to Design and Implementation,* San Francisco, CA: John Wiley & Sons.

Mevarech, Z. R., and B. Kramarski. 1997. “Improve: A Multidimensional Method for Teaching Mathematics in Heterogeneous Classrooms.” *American Educational Research Journal* 34 (2): 365- 394. https://doi.org/10.3102/00028312034002365

Miles, M, B., A. M. Huberman, and J. Saldana. 2014. *Qualitative Data Analysis: A Methods Sourcebook*. London: Sage.

Mills, G. E. and L. R. Gay. 2019. *Competencies for Analysis and Applications*. NY: Pearson Education.

Montague, M. 1992. “The Effects of Cognitive and Metacognitive Strategy Instruction on the Mathematical Problem Solving of Middle School Students with Learning Disabilities.” *Journal of Learning Disabilities* 25 (4): 230–248. https://doi.org/10.1177/002221949202500404

Montague, M., and B. Applegate. 1993. “Middle School Students’ Mathematical Problem Solving: An Analysis of Think-Aloud Protocols.” *Learning Disabilities Quarterly* 16: 19- 32. https://www.jstor.org/stable/1511157

Ohtani, K., and T. Hisasaka. 2018. “Beyond Intelligence: A Metaanalytic Review of the Relationship Among Metacognition, Intelligence, and Academic Performance.” *Metacognition Learning*, 13: 179–212. https://doi.org/10.1007/s11409-018-9183-8.

Özkubat, U., and E. R. Özmen. 2021. “Identifying the Cognitive and Metacognitive Strategies Used by Students with Learning Disabilities and Low and Average Achieving Students During

Mathematical Problem Solving.” *Ankara University Faculty of Educational Sciences Journal of Special Education* 58 (1): 1-38. doi:10.21565/ozelegitimdergisi.736761

Patton, M. Q. 2015. *Qualitative Research & Evaluation Methods.* Thousand Oaks, CA: Sage.

Posamentier, A. S., and S. Krulik. 2008. *Problem Solving Strategies for Efficient and Elegant Solutions: A Resource for The Mathematics Teacher.* Thousand Oaks: Corwin Press

Purnomo, D., T. Nusantara, S. Rahardjo, and S. Subanji. 2016. “Metacognition Process Characteristics of The Students in Solving Mathematics Problems.” *IOSR Journal of Research & Method in Education* 6 (5): 26-35. doi: 10.9790/7388-0605032635

Rickey, D., and A. M. Stacy. 2000. “The Role Metacognition in Learning Chemistry.” *Journal of Chemical Education* 77: 915-920. doi:10.1021/ed077p915

Rofii, A., S. Sunardi, and M. 2018. “Characteristics of students’ metacognition process at informal deduction thinking level in geometry problems.” *International Journal on Emerging Mathematics Education* 2 (1): 89-104. doi:10.12928/ijeme.v2i1.7684

Rosenzweig, C., J. Krawec, and M. Montague. 2011. “Metacognitive Strategy Use of Eighth-Grade Students with and without Learning Disabilities During Mathematical Problem Solving: A Think-Aloud Analysis.” *Journal of Learning Disabilities* 44 (6): 508-520. doi:10.1177/0022219410378445

Schneider, W., and C. Artelt. 2010. “Metacognition and Mathematics Education.” *ZDM Mathematics Education* 42: 149–161. doi:10.1007/s11858-010-0240-2.

Schoenfeld, A. H. 1987. “What’s all the Fuss about Metacognition?.” In *Cognitive science and mathematics education*, edited by A. H. Schoenfeld, 189-215. Hillsdale, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A. H. 1992. “Learning to Think Mathematically: Problem Solving, Metacognition, and Sense-Making in Mathematics.” In *Handbook for Research on Mathematics Teaching and Learning*, edited by D. A. Grouws, 334- 370. New York: MacMillan

Schraw, G., and D. Moshman. 1995. “Metacognitive Theories.” *Educational Psychology Review* 7 (4): 351-371. doi:10.1007/BF02212307

Schunk, D. H. 2012. *Learning theories: An educational perspective*. Boston, MA: Pearson Education Inc..

Scott, B. M., and M. G. Levy. 2013. “Metacognition: Examining the Components of a Fuzzy Concept.” *Educational Research* 2 (2): 120-131. doi: 10.5838/erej.2013.22.04

Serin, M. K., and Korkmaz, I. (2018). “Investigatıon of Cognitive-Metacognitive Behaviors of Primary School 4th Grade Students in Processes of Understanding the Problem and Prediction.” *Adiyaman University Journal of Social Sciences* 10 (28): 131-173. doi:10.14520/adyusbd.327680

Shilo, A., and K. Kramarski. 2019. "Mathematical Metacognitive Discourse: How Can It be Developed Among Teachers and Their Students? Empirical Evidence From A Videotaped Lesson and Two Case Studies. *ZDM Mathematics Education* 51 (4): 625–640. doi:10.1007/s11858-018-01016-6

Shorten, A., and J. Smith. 2017. "Mixed Methods Research: Expanding the Evidence Base. *Evidence-Based Nursing* 20 (3): 74-75. doi:10.1136/eb-2017-102699

Smith, J. M. and R. Mancy. 2018. "Exploring the Relationship Between Metacognitive and Collaborative Talk During Group Mathematical Problem-Solving – What Do We Mean By Collaborative Metacognition?." *Research in Mathematics Education* 20 (1): 14-36, doi:10.1080/14794802.2017.1410215

Sorby S. A., and G. C. Panther. 2020. "Is the Key to Better PISA Math Scores Improving Spatial Skills?." *Mathematics Education Research Journal* 32: 213-233. doi:10.1007/s13394-020-00328-9

Sperling, R. A., B. C. Howard, L. A. Miller, and C. Murphy. 2002. "Measures of Children's Knowledge and Regulation of Cognition." *Contemporary Educational Psychology* 27: 51-79. doi:10.1006/ceps.2001.1091,

Stillman, G., and Z. Mevarech. 2010. "Metacognition Research in Mathematics Education: From Hot Topic to Mature Field." *ZDM Mathematics Education* 42: 145–148. doi:10.1007/s11858-010-0245-x

Susanto, S., and M. Irvan. 2018. "An Analysis of Students' Metacognition Ability Through Jumping Task Strategy to Solve Geometry Problem." *International Journal of Advanced Research* 6 (3): 1375-1381. doi:10.21474/IJAR01/6808

Sutherland, L. 2002. "Developing Problem Solving Expertise: The Impact of Instruction in a Question Analysis Strategy." *Learning and Instruction* 12: 155–187. www.elsevier.com/locate/learninstruc

Swanson, H. L. 1990. Influence of Metacognitive Knowledge and Aptitude on Problem Solving." *Journal of Educational Psychology* 82 (2): 306-314. https://doi.org/10.1037/0022-0663.82.2.306

Teong, S. K. 2003. "The Effect of Metacognitive Training on Mathematical Word-Problem Solving." *Journal of Computer Assisted Learning* 19: 46-55. doi:10.1046/j.0266-4909.2003.00005.x

Veenman, M. V. J. 2017. "Learning to Self-Monitor and Self-Regulate." In *Handbook Of Research On Learning and Instruction,* edited by R. Mayer and P. Alexander, 233–257. New York: Routledge.

Veenman, M. V. J., and D. V. Cleef. 2019. "Measuring Metacognitive Skills For Mathematics: Students' Self-Reports vs. Online Assessment Methods." *ZDM The International Journal on Mathematics Education* 51 (4): 691-701. doi:10.1007/s11858-018-1006-5

Verschaffel, L., F. Depaepe, and Z. Mevarech. 2019. "Learning Mathematics in Metacognitively Oriented ICT-Based Learning Environments: A Systematic Review of the Literature." *Hindawi Education Research International* 2019: 1-19. doi:10.1155/2019/3402035.

Wilson, J. 2001. "Methodological Difficulties of Assessing Metacognition: A New Approach. Australian Association for Research in Education Conference, 1-14. ERIC Number: ED460143. http://www.aare.edu.au/index.htm.

Young, A. E., and F. C. Worrell. 2018. "Comparing Metacognition Assessments of Mathematics in Academically Talented Students." *Gifted Child Quarterly* 62 (3): 259–275. doi:10.1177/0016986218755915

Zhao, N., S. Teng, Y. Li, S. Wang, W. Li, H. Wen, and Y. Mengya. 2019. "A Path Model for Metacognition and Its Relation to Problem-Solving Strategies and Achievement for Different Tasks." *ZDM Mathematics Education* 51 (4): 641-653. doi:10.1007/s11858-019-01067-3