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Examination of Mathematical Errors and Mistakes in Calculus Course

Davut KÖĞCE[1]

Niğde Ömer Halisdemir University

Abstract

This study was conducted to identify mathematical errors and mistakes made by preservice elementary mathematics teachers in the calculus course. To that end, the document analysis method of qualitative research models was used in the research. The sample of the research included a total of 75 preservice teachers who were attending the Department of Elementary Mathematics Education in the Faculty of Education at a public university and taking the calculus I course during the fall term of 2016-2017 academic year. Accordingly, written documents including the participant preservice teachers' papers of interim exams, practice exams, and general exams constituted the data source. The exam papers were scanned with a scanner and transformed into the electronic environment. A content analysis was performed with the data by using MAXQDA 12 qualitative data analysis program, and "data coding" of data analysis techniques was utilized. It was concluded that the preservice teachers made procedural and conceptual errors and mistakes, mathematical errors and mistakes such as recalling generalizations incompletely or incorrectly. Some recommendations were made in light of these results.

Keywords: Preservice mathematics teachers, Calculus course, Mathematical errors, Mathematical mistakes

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[1]Assoc. Prof. Dr., Niğde Ömer Halisdemir University, Faculty of Education, Department of Elemantary Mathematics Education, Niğde, Turkey, ORCID: 0000-0002-3475-2740

Correspondence: kogced@gmail.com

Introduction

Calculus course that mainly examines the functions by utilizing the disciplines of algebra and geometry is an important subject-matter course given in the departments of mathematics education. Mathematical concepts addressed in the calculus course lay the foundation of advanced mathematics used in the disciplines such as mathematics, sciences, engineering, etc. (Ubuz, 1999). Proper understanding and interpretation of advanced mathematical concepts depends on the appropriate learning of concepts given in the calculus course (Gökçek & Açıkyıldız, 2016). In the calculus books, concepts of functions, series, limit, continuity, derivative, and integral are addressed in general (Balcı, 2016; Demir, 2008).

One of the biggest objectives in the calculus course is to examine functions and behaviors of functions. In other words, it is the branch of mathematics engaged in the analysis of functions. Calculus course focuses on examining the image or behavior of a function when its independent variable is infinitesimal or infinitely big. Within this context, students are introduced to the concepts of function, limit and continuity in the first place, and then, several exercises are performed so that they become more competent at operations related to these concepts (Kabaca, 2011). Next, the concept of derivative, conceptually based on limit and continuity, is taught while emphasis is made on the operation of finding the function which is a derivative of a function in the meantime. Finally, concept of integral, also conceptually based on the concept of limit, is taught. In the teaching of integral, firstly, finding an antiderivative for a function is addressed, and then, it is taught under the title “applications of integral” that the concept of integral means a Riemann sum applied under the operation of limit. In summary, Calculus is an important branch of mathematics that include limit, continuity, derivative, integral of functions, and applications of these concepts (Sofronas et al., 2011).

In Turkey, students meet these concepts addressed in the calculus course in their secondary education years for the first time. Thus, they are key mathematical concepts for students who will choose majors related to sciences and mathematics at university. Since understanding of these concepts substantially requires correlating them, it also requires advance thinking skills (Gökçek & Açıkyıldız, 2016). Students have difficulty comprehending these concepts due to their consecutive and abstract nature (Dereli, 2015). Hence, studies are carried out so that students can comprehend the concepts in the calculus course (Barak, 2007). According to Ubuz (1999), tendency to do operations by rote, conceptual incompetence, and the need to improve quality in advance mathematics teaching require studies about concepts in the calculus course.

Answers provided throughout the calculus courses and to the exam questions by students who have succeeded in passing the university entrance exams and enrolled in the department of elementary mathematics teaching show that they make a great number of errors and mistakes in regard to those concepts. There might be several reasons for these errors. One reason might be that high school teachers ignore their actual roles about teaching and perform question solving activities based on rules or formulas to prepare students for the exams. In this case, teachers overlook conceptual learning and give wide coverage to question solving based on procedural learning to prepare students for university entrance exams. In other words, when teachers set it as an objective to prepare students for undergraduate placement exams and lay more emphasis on subjects about possible questions in such exams and teach the subjects in line with rote learning, it might prevent students from learning the mathematical concepts completely. This is supported by the statements of preservice mathematics teachers attending the faculty of education under teacher formation program. As stated by them, mathematics teachers whom they observe at the schools they visit for the Teaching Practice course usually teach subjects in summary and at a fast pace and solve a great number of questions for university entrance exams. This means that teachers conduct instruction based on procedural learning rather than conceptual learning. As a result of learning that ignores the conceptual aspect, students make several errors and acquire misconceptions about mathematical concepts (Gür & Barak, 2007; Koparan, Yıldız, Köğce, & Güven, 2010; Yıldız, Baki, Aydın, & Köğce, 2010; Yıldız, Taşkın, Aydın, & Köğce, 2011; Yıldız, Taşkın, Köğce, & Aydın, 2011). Tall (1992) states that abstract thinking is a prerequisite for transition to advanced mathematical thinking. Although lecturers try to teach in a way that they improve students' conceptual understanding to ensure their transition to advanced mathematical thinking in the undergraduate education, preservice teachers tend to maintain the learning mentality which they have acquired before undergraduate education. Both in the calculus course and the others, whereas preservice mathematical teachers can answer quickly if they know the proper formula or rule for the answer when they are asked questions, they find it difficult to explain the underlying conceptual structure of their operations when they are asked to justify their answers. That is to say, if they cannot recall the formula or rules regarding the answer, they stop finding the solution, or even if they find it, they have difficulty making a conceptual explanation of their solutions. In fact, I observed informally during the analysis course teaching that the mistakes made by some pre-service teachers who could not produce correct answers to the questions included unscientific insights. In the literature, non-scientific conception is commonly called "misconception" (Driver & Easley, 1978). Accordingly, the fact that some of the errors or mistakes made by preservice teachers in the calculus course are scientifically unacceptable warranted the investigation of such mathematical errors. Studies in the literature (Kertil, 2014; Zandieh, 2000) have explored that students have deficiencies in their conceptual understanding although they possess procedural understanding of some of the basic concepts in the Calculus course.

It is seen in the literature that studies have been performed on how undergraduates comprehend certain mathematical concepts addressed by the content of calculus course (Baki & Çekmez, 2012; Delice & Sevimli, 2012; Gökçek & Açıkyıldız, 2016; Göktaş & Erdoğan, 2016; Doruk & Kaplan, 2018). However, no study was observed to have investigated errors and mistakes made by preservice teachers and the difficulties they experience when using the concepts in the calculus course or doing operations. The results to be obtained from this study can also help the development of qualified materials and textbooks related to the calculus course. Thus, this study aimed to identify mathematical errors and mistakes made by preservice elementary mathematics teachers in the calculus course. The following main research problem was asked to that end: “What mathematical errors and mistakes are made by preservice elementary mathematics teachers in calculus course?”

Method

Research Model

This study utilizes the document review method of qualitative research methods which allows for the examination of a certain text or document by digitizing its properties through content analysis (Karasar, 2019). Document review includes the analysis of written and published documents about the subjects to be studied. Document review makes it possible to analyze documents generated in a certain period of time about a research problem or to analyze documents generated by multiple sources and at different times about the relevant subject in a long period of time (Çepni, 2012; Yıldırım & Şimşek, 2018).

Participants

The participants of the research included a total of 75 preservice teachers who were attending the Department of Elementary Mathematics Education in the Faculty of Education at a public university and taking the Calculus I course during the fall term of 2016-2017 academic year. The criterion sampling method was used to determine the participants of the study. In this type of sampling, the study group is determined according to predetermined criteria ((Patton, 2014). The fact that the teacher candidates in the study group were taking the Analysis I course was determined as a criterion.

Data Collection and Analysis

In this study, the answers given by 75 pre-service teachers who took the Analysis I course to the midterm, practice exam and general exam questions were used as the data source. Ten open-ended questions were asked to prospective teachers in each of the midterm, practical and general exams. In other words, 30 open-ended questions were asked in total. These questions were prepared by the instructor in order to measure the knowledge of the pre-service teachers in the concepts of limit, continuity, asymptote, derivative, increasing and decreasing, maximum and minimum, curvature of curves, curve drawings and integral concepts in functions.

The data obtained with the document review were subjected to content analysis of qualitative analysis methods, and data coding of data analysis methods was utilized in the study. It is mainly aimed with the content analysis to explore concepts and relationships that can explain the collected data. In other words, content analysis is a top-tier analysis based on coding. Coding allows for identification and categorization of data. It is accordingly required that the data obtained are first coded, and then, they are logically organized by the codes and themes, which explain the data, are determined. Thus, coding enables the examination and interpretation of data time and again. In this study, 75 participant preservice mathematics teachers' papers of interim exams, practice exams, and general exams were scanned with a scanner and transferred into the electronic environment. The digitalized data were analyzed using the MAXQDA 12 qualitative data analysis program. An answer key prepared by the research for the solution of each question in the documents to be examined in the research was reviewed together with a subject-matter expert, and it was ensured that the expert became familiar with the solutions. To analyze the obtained data in a reliable way, 10 randomly selected exam papers were independently analyzed by the researcher and subject-matter expert categorizing the answers by their similarities and differences (Merriam, 1988; Yin, 1994). The degree of agreement of the coding performed by the researcher and the subject-matter expert was calculated with the formula "Reliability = (Number of categories agreed on) / (Total number of categories agreed and disagreed on)" (Miles & Huberman, 1994). It was concluded that the coding performed independently by the researcher and the subject-matter expert was reliable by 95%. Miles & Huberman (1994) state that inter-rater agreement being 0.70 and above is sufficient for reliability. It was therefore decided that the agreement between the coders was reliable.

The categories generated by the researcher and the subject-matter expert in separate analyses were reviewed by them together; similar categories were clarified, dissimilar categories were discussed, and a consensus was reached through discussion (Merriam, 1988; Yin, 1994). Next, as agreed by the researcher and the expert, the rest of the exam papers would be analyzed by the researcher alone, and at the end of the analysis, the generated codes and themes would be reviewed together with the expert. Accordingly, the remaining exam papers of the preservice teachers were analyzed and categorized by the researcher by their similarities and differences. The codes and themes generated once the data analysis was completed were submitted to the review by the same subject-matter expert and finalized according to the recommendations.

Those codes generated with the content analysis are presented in Table 1 of the findings section along with frequency and percentage values. Moreover, examples for each type of error are presented in Figure 1-14 with images of the actual participant answers.

Results

Mathematical errors and mistakes made by the preservice elementary mathematics teachers in calculus course are given in Table 1.

Table 1. Mathematical Errors and Mistakes Made in Calculus Course

Theme	Subtheme	Codes	f	%
Procedural Errors and Mistakes	Errors and mistakes made in examining the change of functions	Failure to create table of signs by interpreting the data	204	16
		Failure to determine convexity or concavity of the function	125	9.83
		Failure to determine the domain	110	8.65
		Failure to interpret the double roots	98	7.7
		Failure to interpret table of signs	96	7.55
		Failure to determine the intervals where the function is increasing and decreasing	90	7.08
		Failure to determine the asymptotes	63	4.95
		Failure to determine function’s behavior at extreme endpoints of domain	55	4.32
		Failure to determine the points where the function crosses the axes	38	2.99
		Failure to determine cut point of functions	6	0.47
	Making operational errors		98	7.7
	Failure to determine the limit value		84	6.6
	Failure to use mathematical statements (notations) properly		19	1.49
	Failure to do transformations		15	1.18
	Failure to write a special function as a piecewise function		14	1.1
Conceptual errors and mistakes	Overspecification of a concept	Thinking of every function as polynomial function when taking a derivative	42	3.3
		Thinking of integral as taking derivative	16	1.26
		Thinking of limit as taking derivative	14	1.1
		Thinking of infinite as a certain number	3	0.24
		Thinking of value of integral as area under curve	1	0.08
		Taking only the variable when taking the logarithm of both sides of an equation	1	0.08
		Confusing the concepts of indeterminate and undefined	1	0.08
		Considering square root and exact value as equals	1	0.08
	Overgeneralization of a concept	Thinking of a constant as a variable when evaluating a derivative	15	1.18
		Thinking of finding an integral of trigonometric functions as finding an integral of polynomial functions	14	1.1
		Thinking of taking a derivative of trigonometric functions as taking a derivative of exponential functions	2	0.16

	Thinking that a radicand will always be positive	6	0.47
	Thinking of finding an integral of simple fraction statements as finding an integral of logarithmic function	3	0.24
	Thinking of log function as a multiplier when taking the logarithm of the sum of two statements (log(a+b) = loga + logb)	3	0.24
	Ignoring the function when doing an operation on a variable	2	0.16
	Thinking of a fraction as the sum of fractions that consider each term of the statement as a distinct denominator	1	0.08
	Ignoring the degree of function when finding an integral of trigonometric function	1	0.08
	Expanding a statement of which power is rational by using the square of the sum of two terms	1	0.08
	Thinking of function and variable as a multiplier in a function f(x) = f.x	1	0.08
Recalling generalizations incompletely or incorrectly	Confusing a formula or a rule	29	2.28
Total		1272	100

As seen in Table 1, mathematical errors made by the preservice teachers were grouped as procedural errors, conceptual errors, and recalling generalizations incompletely or incorrectly.

Procedural themes were divided into 6 subthemes. As for the rate of procedural errors and mistakes, 69.54% of the preservice teachers were observed to make errors when examining the change of functions. The most observed error among the errors and mistakes made by the preservice teachers when examining the function transformation is failure to create table of signs by interpreting the data (16%). Figure 1 and Figure 2 present the examples of this error and mistake type from the exam papers of two preservice teachers.

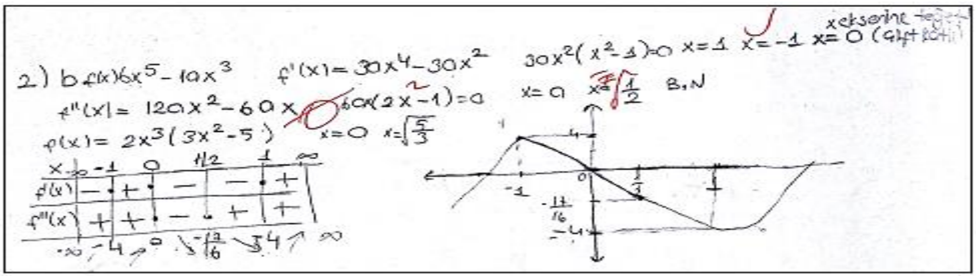

Figure 1. Failure to create table of signs by interpreting the data

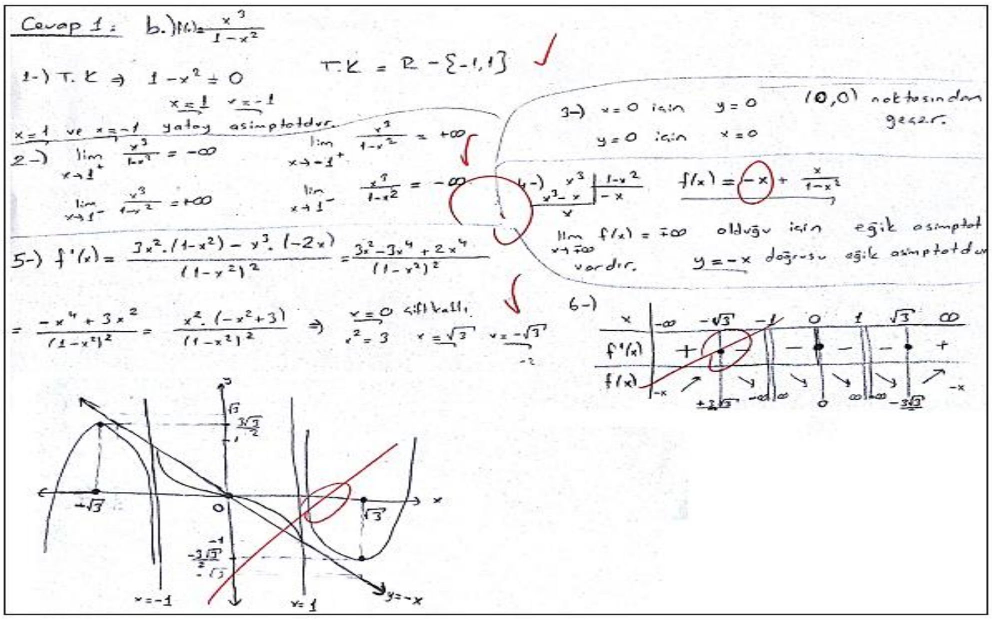

Figure 2. Failure to create table of signs by interpreting the data

Another type of error and mistake made by 9.83% of the preservice teacher when examining the function transformation was failure to determine convexity or concavity of a function. In the exemplary image above (Figure 2), it is also seen that the preservice teacher could not determine convexity or concavity of the function by utilizing the sign of the second derivative.

In this subtheme, another frequent error and mistake made by 8.65% of the preservice teachers was failure to find the domain of a function. Figure 3 presents the exemplary image of this error type from the exam paper of a preservice teacher.

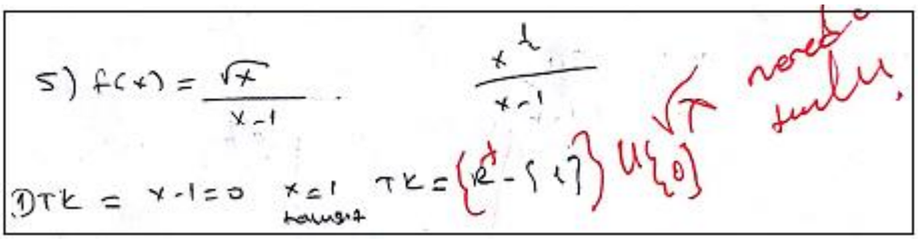

Figure 3. Failure to determine the domain

It was found that 7.7% of the preservice teachers could not interpret the double root. According to Figure 1, although the preservice teacher found the double root among the roots of the first derivative of the function, they could not interpret the double root when creating the table of signs and failed to create the table of signs.

Another error and mistake made by 7.08% of the preservice teachers was failure to find the intervals where the function is increasing and decreasing. Figure 4 presents the exemplary image of this error and mistake type from the exam paper of a preservice teacher.

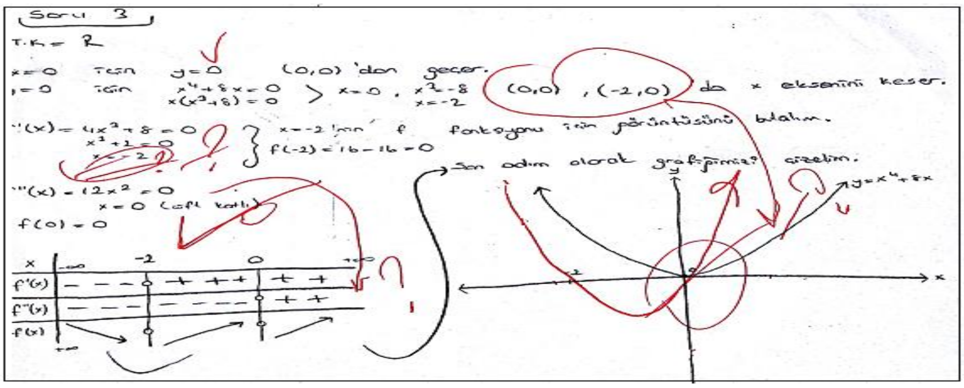

Figure 4. Failure to determine the intervals where the function is increasing and decreasing

Failing to interpret table of signs, and consequently, to draw the graph properly was another error and mistake made by 7.55% of the preservice teachers. Figure 5 shows the exemplary images of this error and mistake type from the exam papers of two preservice teachers.

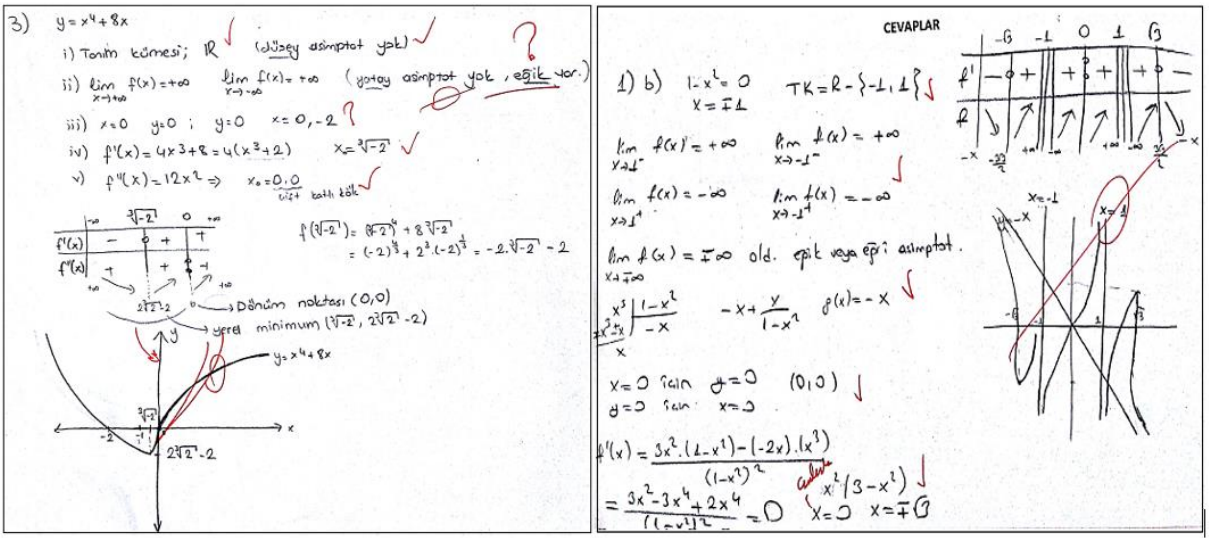

Figure 5. Failure to interpret table of signs

Another error and mistake made by 4.95% of the preservice teachers was failure to determine the asymptotes when drawing a graph. The exemplary images of this error and mistake type from the exam papers of two preservice teachers are presented in Figure 6.

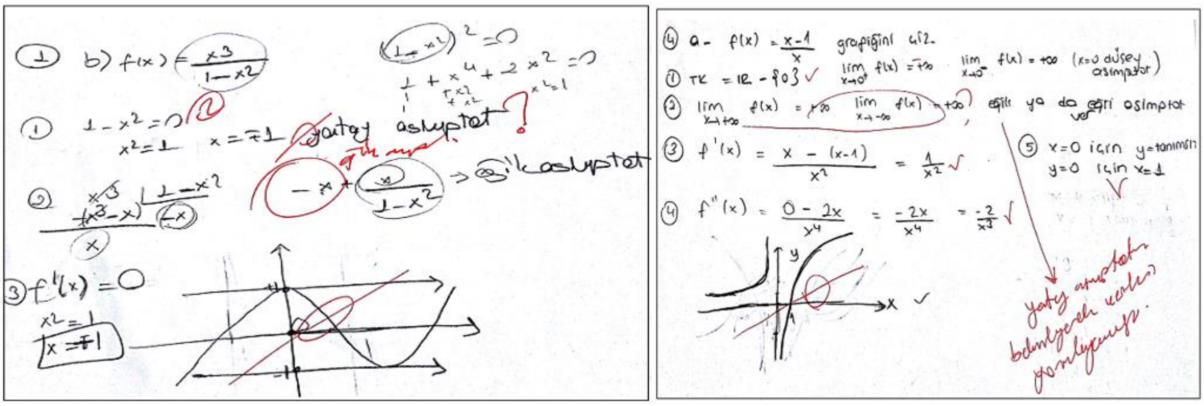

Figure 6. Failure to determine the asymptotes

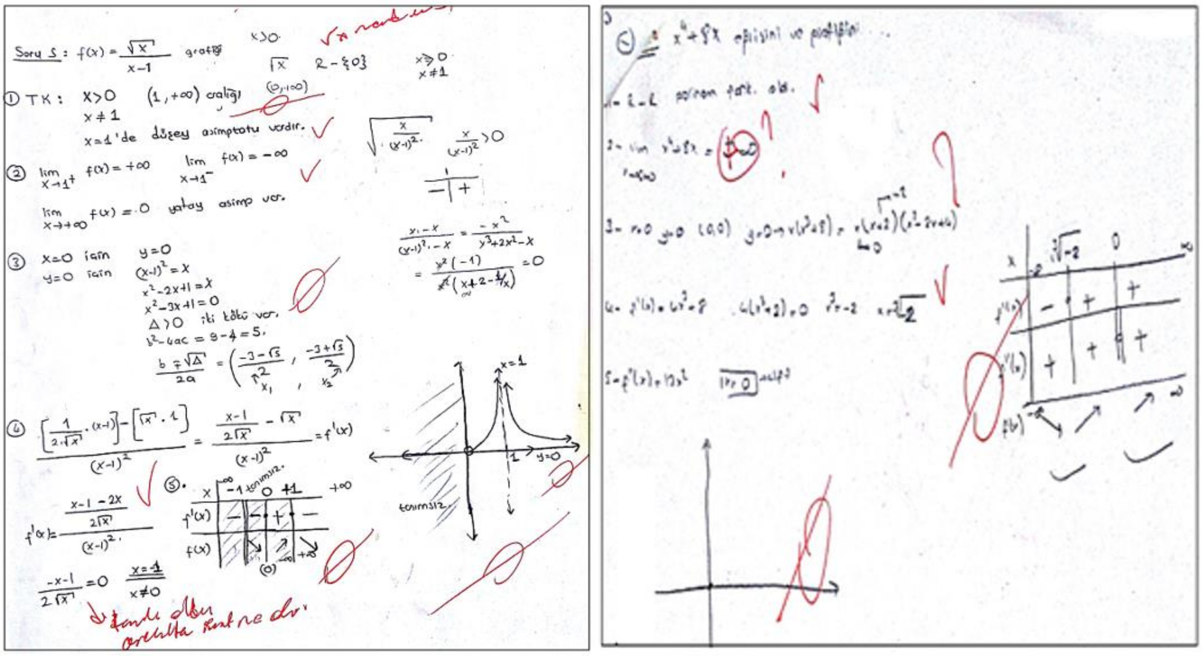

Figure 7. Failure to find function's points of intersection with the axes

Failing to determine function's behavior at extreme points of domain ends was another error made by 4.32% of the preservice teachers. Regarding this error type, as seen in the exemplary image in Figure 6, the preservice teacher could not determine the function's behaviors at extreme points of domain ends when finding the horizontal and vertical asymptotes.

Albeit low, 2.99% of the preservice teachers were found to make errors and mistakes in finding function's points of intersection with the axes. Figure 7 shows the exemplary images of this error and mistake type from the exam papers of two preservice teachers.

Other errors and mistakes under the theme of procedural errors and mistakes included making operational errors and mistakes when solving the questions (7.7%), failure to evaluate the limit (6.6%), failure to use mathematical statements (notations) properly (1.49%), failure to do transformations (1.18%), and failure to write a special function as a piecewise function (1.1%).

Exemplary images for the error in using mathematical statements (notations) properly from the exam papers of four preservice teachers are given in Figure 8.

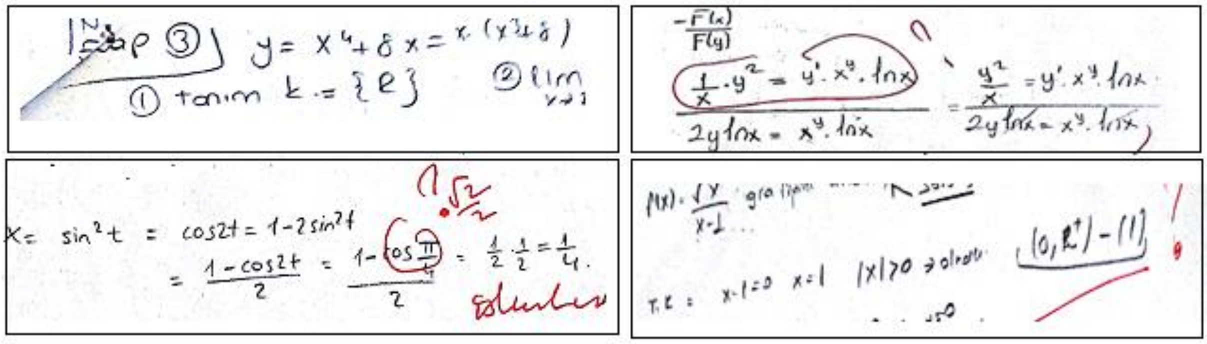

Figure 8. Failure to use mathematical statements (notations) properly

As for the errors categorized under the theme of conceptual errors and mistakes (Table 1), 6.22% of the preservice teachers overspecified the concept and 3.7% overgeneralized the concept.

Although overspecification of concepts had a lower rate among all types of error and mistake, it was found that the preservice teachers made errors and mistakes in 8 different types. The most frequent error and mistake made in overspecification of a concept was thinking of every function as a polynomial function when evaluating a derivative (3.3%), followed by considering finding an integral as evaluating a derivative (1.26%) and thinking of evaluating a limit as evaluating a derivative. For these three error types, Figure 9 and Figure 10 present the exemplary images from the exam papers of three preservice teachers.

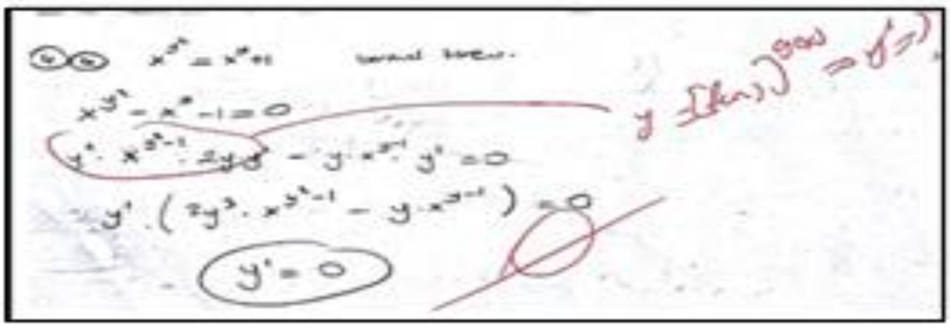

Figure 9. Thinking of every function as polynomial function when evaluating a derivative

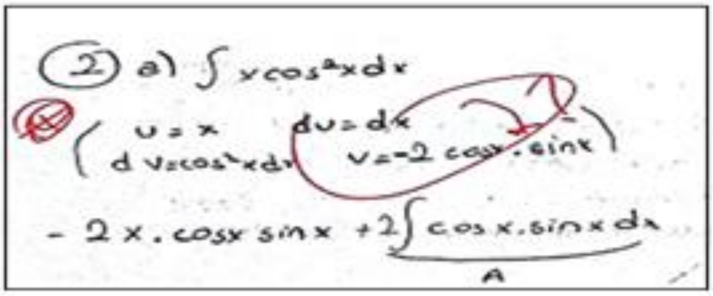

Figure 10. Considering finding an integral as evaluating a derivative

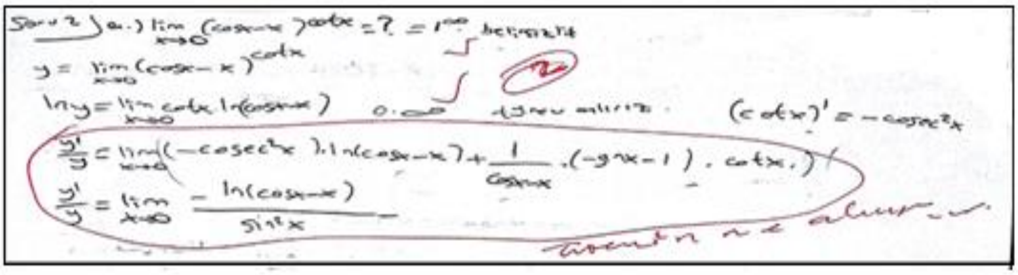

Figure 11. Thinking of evaluating a limit as evaluating a derivative

Despite a low rate for overgeneralization of concepts among all types of error, it was found that the preservice teachers made errors and mistakes in 11 different types. The most frequents errors and mistakes made by the preservice teachers in this type included thinking of a constant as a variable when evaluating a derivative (1.18%) and thinking of finding an integral of trigonometric functions as finding an integral of polynomial functions (1.1%). For these codes, Figure 12 and Figure 13 show the exemplary images from the exam papers of two preservice teachers.

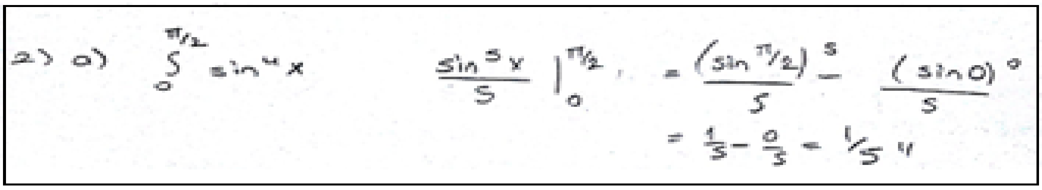

Figure 12. Thinking of finding an integral of trigonometric functions as finding an integral of polynomial functions

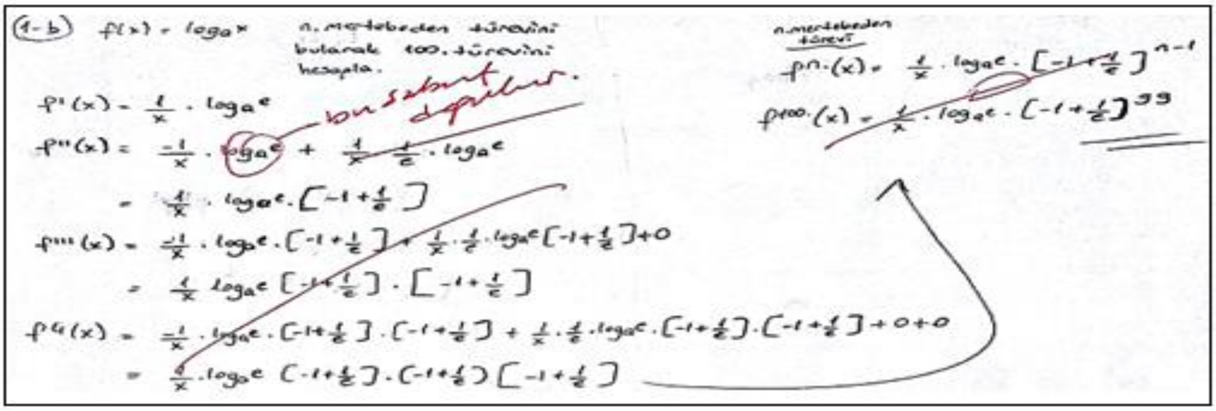

Figure 13. Thinking of a constant as a variable when evaluating a derivative

Following the procedural and conceptual errors and mistakes, another type of error and mistake made by the the preservice teachers was recalling the generalizations incompletely or incorrectly when solving the questions. 2.28% of the preservice teachers made this error and mistake by confusing the formula or the rule in the solution. Regarding these codes, the exemplary images from the exam papers of two preservice teachers are presented in Figure 14.

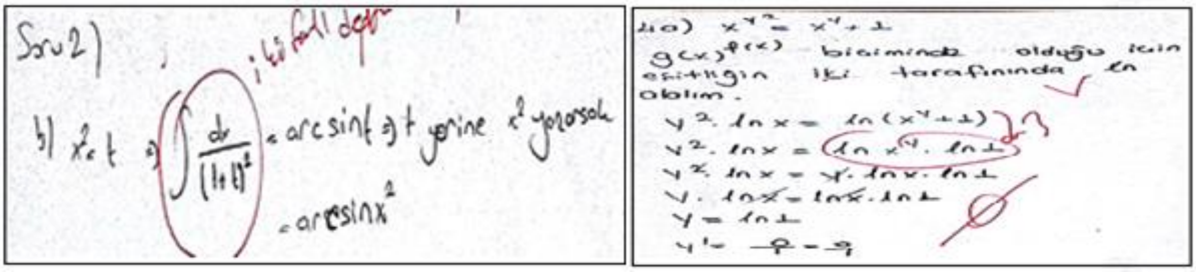

Figure 14. Confusing a formula or a rule

Discussion, Conclusion and Recommendations

In light of the findings achieved in the present study which aimed to identify the mathematical errors and mistakes made by preservice teachers in the Calculus I course, three types of error and mistake were observed: procedural errors and mistakes, conceptual errors and mistakes, recalling generalizations incompletely or incorrectly.

The preservice teachers made several procedural mathematical errors and mistakes when examining the function transformations. The errors and mistakes made by the preservice teachers when examining the change of functions included failure to create able of signs, failure to determine convexity or concavity of the function by examining the sign of the second derivative, failure to find the domain of the function, failure to interpret the double root among the roots of function, failure to interpret the sign of tables they created, failure to find the intervals where the function is increasing and decreasing, failure to determine the asymptotes, failure to determine function's behavior at extreme points of domain ends, and failure to find function's points of intersection with the axes. Based on those errors and mistakes overall, one can argue that the preservice teachers were lacking in procedural knowledge required for drawing the graph of a function. It is reported in the literature that undergraduates make several errors and mistakes when drawing graphs of functions. Those errors and mistakes include drawing the graph of functions as a point (Mevarech and Kramarsky 1997), representing a continuous data point-to-point or discrete data continuously (Brasell and Rowe 1993), and representing the data that should be represented in a single graph by drawing separate graphs (Kramarski 2004). Furthermore, Sierpinska (1992) states that students experience difficulties in subjects such as domain, range, and image of function; inverse function; concept of variable, dependent and independent variables; coordinates; graph, table of function, and function rule. Given the results achieved both in the present study and the literature collectively, procedural and conceptual difficulties experienced by students about the concept of function have an impact on the mathematical errors and mistakes in other subjects of the Calculus course.

In the study, procedural errors and mistakes made by the participants included making operational errors and mistakes when solving the questions, failure to use mathematical statements (notations) properly, failure to do transformations, and failure to write a special function as a piecewise function. Despite being at a lower rate, interestingly, the error "failure to use mathematical statements (notations) properly" was observed for 19 times. Anyone who perform mathematics should be properly using established statements in the field of mathematics. One important reason for this error might be the experiences of preservice teachers prior to the undergraduate education (primary, secondary, and high school). Thus, mathematics teachers should take care to use mathematical concepts or expressions correctly in the teaching process.

The conceptual errors and mistakes observed in the study were grouped under two categories of overspecification and overgeneralization of a concept. Overspecification is the use of a rule, principle or concept by reducing it to a limited comprehension whereas overgeneralization refers to thinking as if a certain rule, principle or concept was taught in other classes, too (Özmantar, Bingölbali and Akkoç 2013). It was observed that the preservice teachers made several errors and mistakes by overspecifying and overgeneralizing the concepts in the solutions. The most frequent errors and mistakes made in overspecification of a concept included thinking of every function as a polynomial function when evaluating a derivative, considering finding an integral as evaluating a derivative, and thinking of evaluating a limit as evaluating a derivative. The prominent errors and mistakes in the category "overgeneralization of a concept" included thinking of a constant as a variable when evaluating a derivative and thinking of finding an integral of trigonometric functions as finding an integral of polynomial functions. It was also found that the preservice teachers made several other errors and mistakes in overspecification and overgeneralization of a concept. There might be many reasons for such errors and mistakes made by the preservice teachers. For instance, they might have chosen to memorize information on each concept rather than learning basic properties of mathematical rules, principles or concepts and correlating the concepts with each other. Indeed, one of the most important errors and mistakes was that the preservice teachers recalled generalizations incompletely or incorrectly as they confused a formula or a rule. Thus, when teaching the Calculus course, subjects should be taught in consideration of such errors and mistakes that could be made by students.

Consequently, how the preservice teachers made several mathematical errors and mistakes, both procedurally and conceptually, indicates that they do not have the adequate procedural and conceptual knowledge on mathematical concepts. This might also affect whether they prove theorems properly and understand the proofs of the data in the Calculus course. Jones (2000) and Weber (2001) state that preservice mathematics teachers and undergraduates find it difficult to provide and understand proofs in undergraduate courses. To overcome such challenges, it is important for lecturers to address concepts and operations by turns during the Calculus courses in consideration that conceptual and procedural knowledge are not independent from each other, and in the contrary, they reinforce each other. Only then, a balance can be struck between conceptual and procedural knowledge.

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