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An Action Research to Eliminate Mistakes in Multiplication and Division Operations through Realistic Mathematics Education[*]

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Abstract

In the current study, it is aimed to determine the activities that need to be done to eliminate the mistakes made by primary school fourth grade students in multiplication and division operations and to present solution suggestions for eliminating these mistakes. The study employed action research, one of the qualitative research methods. The study group was constructed by the criterion sampling method, one of the purposive sampling methods. The study group is comprised of 10 fourth graders attending a primary school in the spring term of the 2016-2017 school year in the city of İstanbul and making similar mistakes. A student information form, clinical interview form and student worksheets were used as data collection tools in the study. Activities prepared in line with the principles of Realistic Mathematics Education (RME) were applied in order to eliminate the mistakes made by the students in the multiplication and division operations. When the mistakes made by the students in the multiplication and division operations were examined, it was revealed that the source of the mistakes was the operational, conceptual and problem situations. During the implementation of RME activities, it was determined that the mistakes of the students started to be eliminated. After the implementation, it was found that the mistakes of the students committed in the multiplication and division operations decreased. Thus, it can be said that RME is an effective application in reducing the mistakes in multiplication and division operations made by students in primary school.

Keywords: Multiplication operation, division operation, realistic mathematics education, mistake, primary school

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Introduction

In order to foster scientific and technological developments, investments need to be made on engineering and computer technologies, which are grounded on mathematical knowledge and skills. Mathematics education has an important place starting from primary school in training individuals with advanced mathematical knowledge and skills. For this reason, effective teaching of mathematics in primary school is necessary (Baykul, 1992). In order for this process to be carried out effectively in primary school, it has become necessary to identify the problems that students experience during the development of their mathematical skills and to produce solutions for these problems. The source of the problems experienced during the development of mathematical skills is to have students engage in activities focused on the development of procedural knowledge rather than having them make sense of mathematical knowledge based on daily life experiences in mathematics lessons (Uça & Saracaloğlu, 2017). Since the source of mathematical knowledge is daily life problems, applications made for the development of mathematical knowledge and skills should be intertwined with real life and procedural information should be given based on the concept. In this way, it will be easy for students to make sense of mathematical knowledge and skills, they will create their mathematical operations in a meaningful way in daily life, and it will be easier to eliminate problems related to mathematical knowledge and skills.

Starting from pre-school education, providing mathematics education effectively throughout one's education life has an important place in shaping the future of people. While the development of mathematical knowledge and skills in the mathematics education process will create a productive future, failure to develop them will create a stable future (National Council of Teachers of Mathematics [NCTM], 2000). The effective and efficient conduct of mathematics education will be possible by improving the mathematical knowledge and skills of students with low academic success in the field of mathematics. Students with low mathematics achievement are also expressed as students having difficulties in learning mathematics. The National Joint Committee on Learning Disabilities [NJCLD] (2001) identified mathematics difficulty as a sub-difficulty of learning, and expressed it as students' having problems in perceiving mathematical concepts such as place value and time, keeping mathematical information in mind, and organizing numbers and problems. Mathematical learning difficulty is a comprehensive concept that includes misconceptions and mistakes. Misconception is expressed as the perception or understanding that is far from the view on which the experts agree on a subject (Zembat, 2008). Misconception can also be expressed as information that prevents the teaching and learning of concepts that are proven to be true by science and that are contrary to scientific facts formed as a result of personal experiences (Keçeli, 2007). Misconceptions are systematic mistakes that arise from the comprehension of information that has been proven to be true. Misconception, which is a conception that systematically encourages mistake, is closely related to mistake. It has been revealed in many studies that students make mistakes as a result of misconceptions (Barmby, Bilsborough, Harries & Higgins, 2009; Cockburn & Littler, 2008; Harris, 2001; Nesher, 1987; Oliver, 1989; Radatz, 1980; Sadi, 2007). While misconceptions are the source of mistakes, not every mistake arises from a misconception (Yenilmez & Yaşa, 2008). Mistake can be expressed as mistakes in responses and misconceptions can be expressed as conceptual barriers that prevent learning (Yılmaz, 2011). In order for an idea put forward by the student to have a misconception, the student must meet three conditions: his/her thought is not in accordance with real science, he/she must present reasons to adopt this wrong idea and he/she is sure of his/her own answers and explanations (Eryılmaz & Sürmeli, 2002). If the student explains that his/her mistakes are correct with the reasons and makes these explanations confidently, it can be said that he/she has a misconception. Identifying and eliminating the misconceptions that are among the reasons for the mistakes made by students and the sources of these mistakes will increase the student's success in mathematics.

In primary school mathematics education, there are basic arithmetic skills that include addition, subtraction, multiplication and division, which are expressed as basic four operations (NCTM, 2000). Learning the four-operation skills without mistakes in primary school has an important place in gaining the mathematical knowledge and skills necessary for success in mathematics. In order for operations and concepts to have mathematical meaning, a relationship must be established between them (Pearson & Somekh, 2003). It is a natural process for students to make mistakes when there is no connection between operations and concepts (Van de Walle, Karp & Bay-Williams, 2014). For this reason, it is important to identify and correct the mistakes made in the four operations so that students do not have problems in the mathematical learning process. Research on primary school students' mistakes in the four operations has focused on identification (Ashlock, 1994; Brown & Burton, 1978; Chick & Baker, 2005; Cotton, 2010; Engelhardt, 1977; Önal & Aydın, 2022; Sadi, 2007; Thompson & Bramald, 2002) while the research on the elimination of mistakes is not sufficient. It is necessary to know the appropriate strategies to eliminate the mistakes because they will negatively affect the next learning of students (Ashlock, 2002; Ben-Hur, 2006; Ojose, 2015).

In the elimination of mistakes in the four operations, it is important to know the strategies suitable for eliminating mistakes and to know the mistakes made in the implementation of these strategies. It is seen that the mistakes made in the four operations at the primary school education level are made more especially in the multiplication and division operations. According to Nures and Bryant (2008), the reason for teaching multiplication and division after addition and subtraction is because these operations seem more complex and difficult. Ayvaz (2010) defined the division operation as the operation that children have the most difficulty in understanding, both semantically and operationally among the four operations. Due to the relationship between multiplication and division, the source of children's having difficulty and making mistakes in division can be expressed as multiplication. Burns (2007) stated that children should understand the relationship between multiplication and division in order to be more successful in division and not to make mistakes.

In order to perform multiplication and division operations effectively, it is necessary to know addition and subtraction operations. It is important to know the mistakes made in multiplication and division operations for the development and learning of the four operations in children. Harris (2001) classified students' mistakes made in multiplication and division operations. Mistakes made multiplication operations include the confusion experienced by children between the "X" and "+" signs, adding instead of multiplying, writing the carry by directly adding, forgetting the carry, adding the number to the digit by including the carry, multiplying the tens digit with the carry, not being able to create a new digit, mistakes made in multiplying by "0", writing missing digits in multiplication, place value problem, not being aware of the need for cross multiplication, performing adding operations instead of multiplication operations and mistakes made in adding "0" to 10 and the multiples of 10. The mistakes made by children in division operations include confusing "÷" with "X", mistakes in division by "1", dividing small numbers into large numbers, confusing subtraction and multiplication with division, performing subtraction "-" in division by "0", dividing a number by "0" and showing the number as a result, forgetting to move on to the next digit, starting the operation from the right, not from the left, forgetting to add the remainder to the next digit, forgetting the remainder, forgetting to add the value "0" to the division, forgetting to subtract, not adding "0" to the answer, mistakes in multiplication within the division operation and mistakes in subtraction within the division operation. Mistakes in multiplication usually occur in multiplication by "0" and "1" and in shifting digits when multiplying two-digit and two-digit numbers (Bamberger, Oberdorf & Schultz Ferrell 2010; Cockburn & Littler, 2008; Engelhardt, 1977). In the division operation, the mistakes that students usually make are starting the operation from the right, not from the left, mistakes made in the use of the number "0" and in carrying over from one digit to another.

Questions such as how to learn mathematics better, how to increase mathematical knowledge, and how to develop mathematical skills have revealed that knowledge cannot be obtained passively, but that knowledge will be constructed as a result of the learner's own activities. Mathematics teaching should be carried out in a learning environment where students take an active role in the process, learning mathematics is enjoyable for the learner, and a positive attitude towards mathematics is fostered (Uça & Saracaloğlu, 2017). Realistic Mathematics Education (RME), which aims to develop students' formal mathematical knowledge starting from real life situations, is one of the most effective learning theories that increase students' interest in mathematics education and help them learn mathematical concepts and generalizations in a meaningful way (Treffers, 1991). This approach, introduced by the Dutch mathematician Hans Freudenthal, is used in the mathematics teaching process in many countries. According to Freudental, mathematics is a human activity connected with reality; thus, it should be related to society (Zulkardi, 2000). Mathematics should be associated with reality, be close to children's lives and contain human values. Mathematics should not remain as a human activity; it should be influential and usable in people's lives. Since mathematics will affect people's lives, teaching should take place within a contextual learning and teaching process (Theodora & Hidayat, 2018). In RME, students should learn by developing and applying concepts and tools that are meaningful to them according to the problem situations in life (Bakker, 2004).

RME emphasizes that the subject should be meaningful and natural for students. The formal structure of mathematics reflects the content to the extent that it is real in students' minds (Van den Heuvel-Panhuizen, 2000). In RME, it is not started by using abstract formulas, symbols, rules and definitions; instead, it is aimed to learn in practice by creating concrete situations (Wubbels, Korthagen & Broekman, 1997). In RME, the learning process is organized by establishing an organized deductive structure (Ünal & İpek, 2009). Freudenthal argues that learning mathematics in children will begin with sense-making and that sense-making should be taken as a basis at every stage (Altun, 2006). On the basis of theorem developed by Freudenthal (1968) on mathematics education lays the concept of "mathematization" in which mathematical activities are arranged in accordance with the mathematical understanding and grade levels of the students in relation to the situations they may encounter in daily life. Treffers (1978) divided mathematization into two categories: "horizontal mathematization" and "vertical mathematization". Horizontal mathematization refers to the elicitation of all mathematical tools and the selection of the appropriate ones to be used in the organization and solution of daily problems (Gravemeijer & Doorman, 1999). Vertical mathematization is defined as the process of rearranging the mathematical system. It is also defined as the process of reaching high-level mathematics, which aims to reveal conceptual relationships using symbols (Altun, 2006). Freudenthal (1991) stated that there is no clear line between horizontal and vertical mathematization, that they can take place at every stage of mathematical activities and that the person will decide which one to use and where. In RME, the student's learning of mathematics should occur as doing mathematics. The student should reach the required information himself/herself as a result of the problem solving activity. If there is no real situation for mathematics, an imaginary situation must be created. Thus, in a suitable environment, the child can be engaged in the act of mathematization (Altun, 2012). In the transfer of mathematical subjects, some principles should be taken into account while performing the mathematization process.

The principles of mathematics teaching in RME are different from the ones in the traditional way of teaching mathematics but there is some similarity in content. Since RME, which is an approach to teaching and learning mathematics, has its own characteristics, its principles are different from the ones of the traditional way of teaching mathematics. In the development of the teaching design for RME, the principles defined by Gravemijer, Cobb, Bowers and Whitenack (2000) and Gravemijer (1994) were updated and the basic principles of directed re-discovery, instructional phenomenon and developing models were determined. The principles of how teaching should take place were established by Treffers (1987). Treffers (1987) stated five principles: factual exploration through contexts, making connection with vertical materials, students' own sense-making and products, interactive teaching and establishing connections between the stages of learning. These principles developed by Treffers (1987) were re-expressed by Van den Hauvel-Panhuizen and Wijers (2005) as six principles: activity, reality, level, establishing connections between activity areas, interaction and guidance, which include the learning and teaching process of students. In line with these principles, the learning and teaching process was planned and implemented.

Mathematics is a system created mentally by humans. This is a sign that mathematics is abstract, so abstract concepts are difficult to acquire (Gür, Hangül & Kara, 2014). Since mathematical concepts are abstract concepts, students learn through concretization (Ernest, 2010). In order to eliminate mistakes that can occur while learning the four operations conceptually in primary school, meaningful real-life-related problem situations should be created (Barnes, 2004; Barnes, 2005) or concrete tools should be included in a visually enriched environment (Flowers, Green & Piel, 2008; Çilingir & Dinç Artut, 2016; Çilingir Altıner & Dinç Artut, 2017). Importance should be attached to studies that prioritize the way they perceive the concepts of multiplication and division and the strategies they use in primary school. While especially student-centred approaches are adopted, it is necessary to understand how children think about these concepts and what kind of mental processes they use in problem situations they encounter (Carpenter, Fennema, Franke, Levi & Empson, 1999). Since better and permanent learning will be accomplished by establishing a connection with real life in the acquisition of operational skills, the mistakes made by the students will be eliminated. It can be said that the activities prepared in line with the principles of RME will be effective in establishing a relationship between the situations that students encounter in daily life and their learning. Realistic Mathematics Education provides students with concrete experiences from the real world and enables them to learn concepts in a meaningful way by increasing interaction in the classroom environment. In this connection, in the current study, it is aimed to determine the activities that should be done to eliminate the mistakes made by primary school fourth grade students in multiplication and division operations and to produce solutions to eliminate these mistakes. The problem statement of the study was determined as evaluating the effectiveness of RME in eliminating the mistakes made by primary school fourth grade students in multiplication and division operations.

Method

Research Model

Since the aim of the study was to find solutions to the problems that might arise in the process and the application processes and the research were carried out together, action research, one of the qualitative research designs, was adopted in the study. Action research was preferred in the study because it does not aim to generalize and is suitable for intervention in the process. Action research should be systematic, not begin with a response, include planning, be based on regular observation, be simple or detailed, be embedded in theory, not be a quantitative research, not present limited findings (Johnson, 2015). In addition, action research is learning by doing and experiencing, which includes defining a problem, making efforts to solve the problem, seeing the success of the efforts, and finding new solutions if the efforts are unsuccessful (O'Brien, 2001). Action research is a research design used to improve the skills of individuals in the field of education. Since the current study was carried out with the aim of improving the operational skills of students by eliminating their mistakes made in multiplication and division operations, action research was employed in the study.

Study Group

In the determination of the study group of the study, first a primary school located in a region with a low socio-economic status in the city of Istanbul was determined. The mistake detection form for operations was administered by the researcher to 175 fourth grade students attending the primary school in the spring term of the 2016-2017 school year. After the application, the study group was determined by using the purposive sampling method. The study group was formed by taking into account the following criteria: the subjects have already been taught in the lessons before the application and the student made mistakes, the students do not have developmental problems, the students do not have any reports by the Guidance Research Centre, the parents and teachers of the students are willing to work, and the students made similar mistakes according to the mistake detection form. The mistakes made by the students in the study group were determined by taking into account the mistakes they made in the error detection form and the answers in the notebook and in-class worksheets. In line with these criteria and expert opinions, 12 students were determined, and since 2 students did not continue in the process, the research was concluded with 10 students, 5 girls and 5 boys.

Data Collection Tools

In the collection of the data, the student information form, the mistake detection form for the four operations, the audio recordings, the student worksheets and the clinical interview form for the four operations were used. The student information form was prepared in order to determine the demographic characteristics of the student. The error detection form for the four operations was used in the determination of the study group and consists of four parts to determine the errors for addition, subtraction, multiplication and division operations. The audio recording was taken before and after the study to prevent data loss. Student worksheets are prepared for the purpose of checking the action plan and the student and determining the achievement of the action research goal. The worksheets were prepared in line with RME and were used to reveal the development of the student in the process.

The clinical interview form for the four operations consists of 18 questions; 3 for the addition operation, 4 for the subtraction operation, 4 for the multiplication operation and 7 for the division operation. In the mistake detection form, it was determined that the fourth grade students made fewer mistakes in addition and subtraction than in multiplication and division operations. In this connection, more questions about multiplication and division operations were included in the clinical interview form as more mistakes are made in these questions. Therefore, the current study focused on the mistakes made in multiplication and division operations and their elimination. Clinical interview questions were administered to the students before and after the application. The clinical interview was preferred because it could reveal the knowledge structures and thinking processes of the students about the subject, have more flexible questions and allow high interaction. With the applied form, it was aimed to make sense of the mistakes made by the students in the multiplication and division operations. The prepared form was distributed to the students in printed form, the questions in the form were solved by the students, questions were asked to the students when they made mistakes and they were recorded. The students were asked such questions as “How did you solve this question, can you solve it again but loudly this time?”, “How did you continue with the operation?”, “What did you understand from this question?”, “How should this problem be solved?” The students were given the opportunity to think aloud and they were allowed to express themselves loudly. The clinical interviews conducted with 10 students before the application lasted 94 minutes and 7 seconds, and this time was 35 minutes 56 seconds after the application.

Application Process

Since action research includes a dynamic process and has a systematic structure, the data collection process was continuous. Before the application, the personal information of the students in the study group was obtained and clinical interviews were conducted. During the application process, activities developed in line with the RME principles to eliminate mistakes in multiplication and division operations were implemented, and during the implementation of the activities, the students reflected their products on the worksheets. The activities developed in line with RME included not only multiplication and division but also addition and subtraction operations. The reason for this is that most of the mistakes made in multiplication and division are based on the mistakes made in addition and subtraction. After the application, with the clinical interview questions, the students' state of making mistakes in the multiplication and division operations was revealed. All the clinical interviews before and after the application were recorded.

In order to determine the study group during the research process, 175 students were administered the mistake detection form for four operations between 20 and 22 March 2017 in line with the criteria determined at the primary school fourth grade level. The clinical interview form was administered between 28 and 31 March 2017 before the application and between 23 and 31 May 2017 after the application. The clinical interviews with the students were carried out on the specified dates during the appropriate class hours. The RME activities, on the other hand, were administered to the study group in two class hours on Tuesdays and Thursdays between April 4 and May 18, 2017. The application process took 7 weeks, 4 hours a week, 28 hours in total. In line with the RME principles, 2 activities for addition and subtraction, 4 activities for multiplication and division each were prepared and implemented. The reason why the application was carried out in the spring semester was that they had already finished the topics related to multiplication and division. The activities prepared in line with the principles of RME were conducted in the library of the school with the group formed by bringing together the students who made similar mistakes during the lesson.

Data Analysis

The collected data were analyzed using qualitative data analysis methods in line with the sub-purposes of the study. Each student was analyzed individually in line with the clinical interview questions in order to determine the mistakes made by the students in the multiplication and division operations before and after the RME application. The students' multiplication mistakes were determined according to low (0 and 1 mistake), medium (2 mistakes) and high (3 and 4 mistakes) levels over the four multiplication questions in the clinical interview. In the division operation, low (0, 1 and 2 mistakes), medium (3 and 4 mistakes) and high (5, 6 and 7 mistakes) levels were determined over the seven division questions in the clinical interview. In line with these levels, the developmental status of the students in multiplication and division operations was revealed. The change that occurred in the students during the application process was analyzed individually on the basis of the worksheets. Credibility was tried to be established by providing long-term interaction between the researcher and the students, collecting in-depth data, obtaining confirmation from the participants and taking expert opinion on the research process. As a result of the expert review, the content validity coefficient was found to be .76 for the clinical interview form and .78 for the mistake detection form. Transferability was ensured by describing the findings and the study group in detail.

Results

In this part of the study, the mistakes made by the fourth grade students in the multiplication and division operations were determined, the development of the students regarding these mistakes during the application process was revealed, and lastly, the mistakes in the multiplication and division operations after the application were revealed.

The mistakes made by the students participating in the study before the RME activities regarding the multiplication operations are shown in Figure 1.

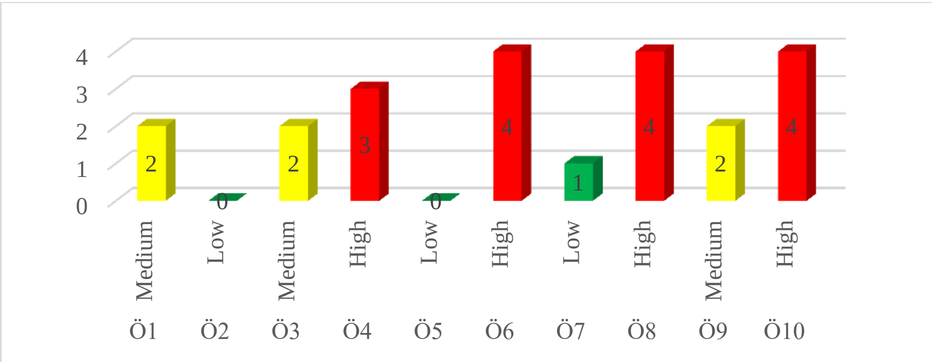

Figure 1. State of the students regarding the mistakes they made in the multiplication operations before the RME application

As can be seen in Figure 1, when the level of mistakes made by the students regarding the multiplication operations before participating in realistic mathematics education was examined, it was found that three students made mistakes at the "low" level, three students made mistakes at the "medium" level and four students made mistakes at the "high" level. Two of the students who were found to be at the "low" level made "0" mistake and the other made "1" mistake. Three of the students who were found to be at the medium level made "2 mistakes". One of the three students who were found to be at the high level made "3" mistakes and the other three students made "4" mistakes. The mistakes made by the students in the multiplication operations were examined individually, and examples of the mistakes made by the student coded S6, who was found to be at the "high" level, and sections from the interview conducted with him/her are given below.

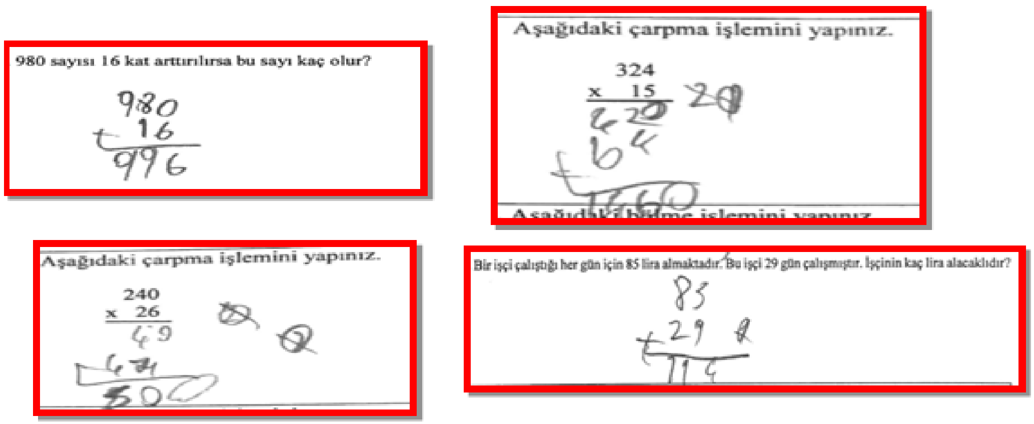

Figure 2. Mistakes made by the student coded as S6 in the multiplication operations before the RME application

The four mistakes made by the student coded as S6 in the multiplication operations are shown in Figure 2. When the first mistake made by the student was examined, it was revealed that the student performed an addition operation instead of a multiplication operation. A mistake was made because he/she thought that the addition operation should be done from the expression "increase" in the question. In addition, a mistake was committed because the word "times" did not connote the multiplication operation. Regarding the solution of the first question in which the student made a mistake, the student said "Addition if it is increased, my teacher." The second and third mistakes made by the student in the multiplication operations are due to the fact that the multiplication of the numbers was performed incorrectly, although the rule of multiplication was known. Regarding the second and third questions in which mistakes were made, the student made the following explanation about his/her solution; "I can't explain, teacher, I don't know how I did it." The fourth mistake made by the student regarding the multiplication operation stems from the fact that he/she was not aware of the necessity of performing a multiplication operation in the case of a problem involving multiplication. While the problem required multiplication, the student performed addition due to his/her prior learning and because it was easy for him/her. Regarding the fourth question in which the student made a mistake, the student said, "Teacher, I didn't understand this part either, so I thought I'd add it."

The mistakes made by the students participating in the study regarding the division operation before the RME activities are given in Figure 3.

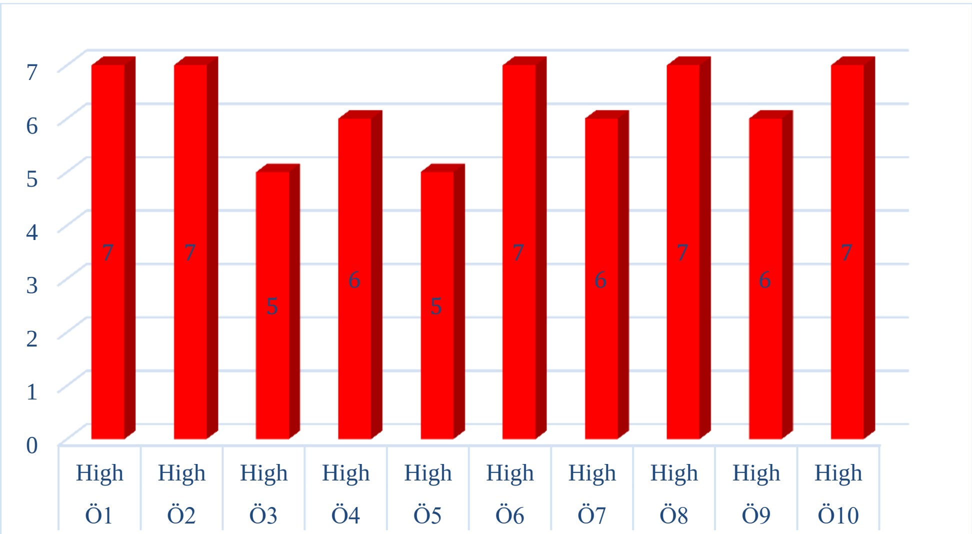

Figure 3. State of the students regarding the mistakes they made in the division operations before the RME application

As can be seen in Figure 3, when the level of mistakes made by students regarding the division operations before participating in realistic mathematics education was examined, it was revealed that ten students who participated in the study made mistakes at the "high" level. Two students at the high level made "5" mistakes, three students made "6" mistakes and five students made "7" mistakes. The mistakes made by the students in the division operations were examined individually, and examples of the mistakes made by the student coded S2, who was found to be at the "high" level, and sections from the interview conducted with him/her are given below.

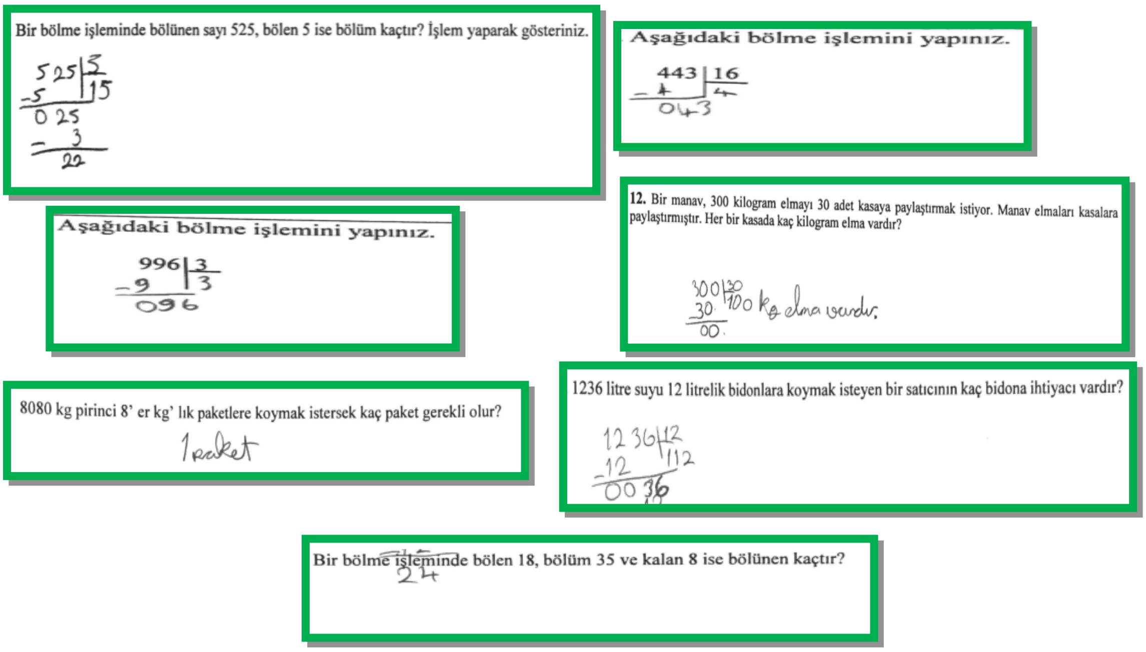

Figure 4. Mistakes made by the student coded as S2 in the division operations before the RME application

In Figure 4, seven mistakes made by the student coded as S2 in the division operations are given. The student made an operational error in adding "0" after starting the process for the first time. Regarding the solution of the first question in which the student made a mistake, the student said, "I guessed that it would be correct when I did division. There is one 5 in 5 but not in 2 thus when we took down 2 we had to remove 0." The second mistake is due to the student searching for 16 inside 4 (the dividend within the divisor). Regarding the second question in which the student made a mistake, the student explained his/her solution as follows; "There are 4, 4 times in 16, I subtracted 4 from 4 and it became 0, I took down 4 and then 3 and I completed the operation." When the third mistake was examined, it was seen that the first step of the division operation was performed, but the next steps could not be performed, so a mistake occurred. Regarding the third question, where the student made a mistake, the student made the following explanation, "There are 3 times 3 in 9, 3 times 3 is 9, 9 is subtracted from 9 and it is 0, I took down 9 and 6, and I completed the operation.” In the fourth mistake, the problem situation was transferred to the operation correctly, but the mistake was made as the number “0” was added to the quotient section although it should not have and thus an operational mistake occurred. Regarding the fourth question, where the student made a mistake, the student made the following explanation, “I got stuck a little bit, but I tried to do it. I divided 300 by 30, there is one 30 in 30, I took down 0 from above and added two 0.” The fifth mistake made by the student is due to his/her misunderstanding the question. Regarding the fifth question, where the student made a mistake, the student made the following explanation, “I was going to do division, but it found it too difficult.” In the sixth mistake made by the student, although the conversion of the problem to the division operation was correct, an operational mistake occurred because the second step of the operation was incorrect. Regarding the sixth question, where the student made a mistake, the student made the following explanation, “I divided 1236 by 12. There is one 12 in 12. I subtracted 12 once out of 36 and got 24. There are 3 12s in 24. The answer is 112.” The seventh mistake is due to the student’s not answering the question. The student explained why he/she did not answer this question, “I did not understand. I couldn’t figure out where the quotient, divisor and remainder would be.”

Examples of the answers given by the student coded as S6 on the worksheets to the questions related to the multiplication operation during the application of RME activities are given in Figure 5.

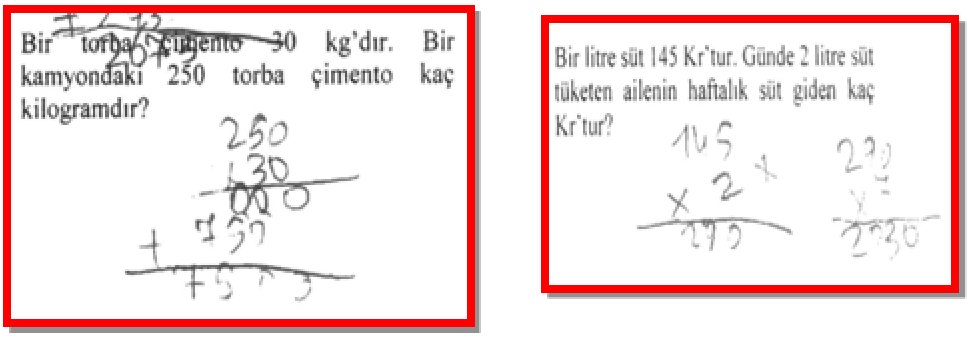

Figure 5. Sample solutions from the worksheets of the student coded as S6 regarding the multiplication operation

Before the research process, the student coded as S6 was making mistakes in the questions involving the concept of “times” in multiplication, making operational mistakes and making mistakes in questions requiring multiplication. The sample questions obtained from the worksheets on the multiplication operation during the application process of the RME activities of the student coded as S6 are given in Figure 5. When the worksheets of the student are examined, it is seen that the mistakes made before the RME have disappeared. It can be said that there is a decrease in the mistakes made by the student in the multiplication operation as a result of the activities prepared in line with RME.

Examples of the answers given by the student coded as S2 on the worksheets to the questions related to the division operation during the application of RME activities are given in Figure 6.

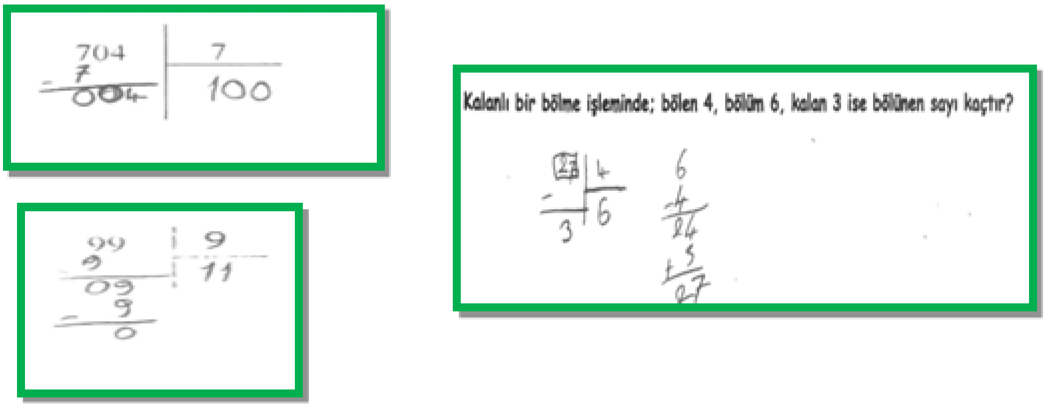

Figure 6. Sample solutions from the worksheets of the student coded as S2 regarding the division operation during the application of RME

Before the research process, the student coded as S2 was making mistakes in the division operation such as adding "0" to the quotient, not being able to continue the division, not knowing the concepts related to division, and problems involving division. The sample questions obtained from the worksheets on the division operation during the application process of the RME activities of the student are given in Figure 6. When the questions in the worksheet are examined, it is seen that the student can add “0” to the quotient in the division operation, continue the division, use the concepts related to division appropriately and perform the appropriate operations. It can be said that there is a decrease in the mistakes made by the student in the division operation as a result of the activities prepared in line with RME.

The mistakes made by the students participating in the study in the multiplication operation after the RME activities are shown in Figure 7.

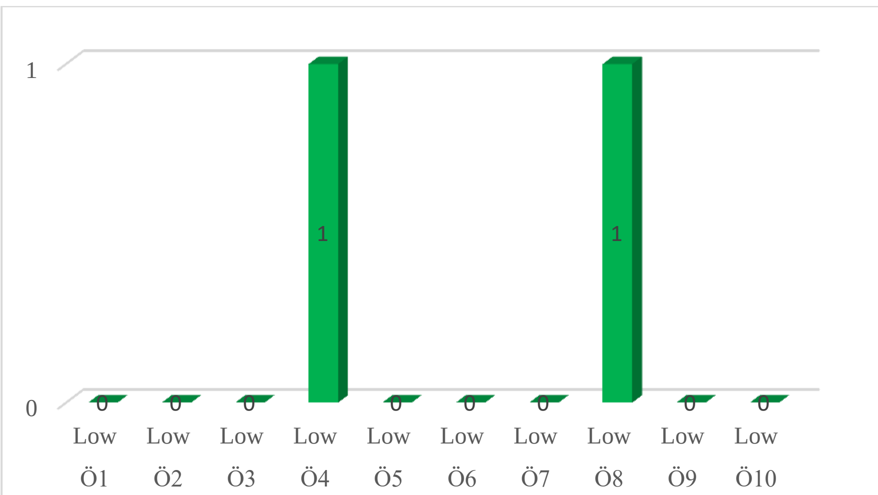

Figure 7. The state of the students in terms of the mistakes they made in the multiplication operations after the application of RME

As can be seen in Figure 7, when the level of the mistakes made by the students in the multiplication operations were examined after the realistic mathematics education, it was found that the ten students who participated in the study made mistakes at the "low" level. While eight low level students made zero mistakes, 2 students made 1 mistake. When the mistake of the student coded as S4 in the mathematic operation was examined, it was seen that he/she made the mistake as he/she perceived the problem situation as requiring addition rather than multiplication and explained his/her understanding as follows; "A worker gets 85 liras a day and then in order to calculate how much he gets in 29 days I add 85 to 29." When the mistake of the student coded as S8 was examined, it was determined that he/she made an operational mistake. On the other hand, it is seen that the student coded as S6 did not make any mistakes in the multiplication operation after the application as could be seen from his/her worksheets.

The mistakes made by the students participating in the study regarding the division operations after the RME activities are given in Figure 8.

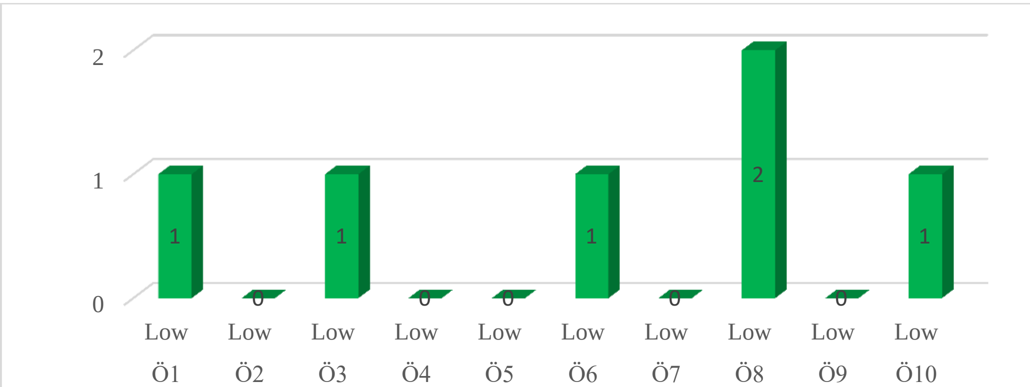

Figure 8. State of the students regarding the mistakes they made in the division operations after the RME application

As can be seen in Figure 8, when the level of the mistakes made by the students regarding the division operations after the realistic mathematics education was examined, it was found that the ten students who participated in the study made mistakes at the "low" level. While 5 students at low level made zero mistakes, 4 students made 1 mistake and 1 student made 2 mistakes. The source of mistake made by the students coded as S1 and S6 who made 1 mistake in the division operation is operational while the source of the mistake made by the students coded as S3 and S10 is due to their not being able to answer the question. The first mistake of the student coded as S8, who made two mistakes, is due to his/her not knowing the concepts of division, and the second mistake is due to his/her not being able to answer the question.

Table 1 shows the change in the mistakes made by the students to whom RME activities were applied in the multiplication and division operations before and after the application.

Table 1. Distribution of the students' mistakes in the multiplication and division operations before and after RME

Student	Multiplication Operation		Division Operation	
Code	Before application	After application	Before application	After application
S1	2	0	7	1
S2	0	0	7	0
S3	2	0	5	1
S4	3	1	6	0
S5	0	0	5	0
S6	4	0	7	1
S7	1	0	6	0
S8	4	1	7	2
S9	2	0	6	0
S10	4	0	7	1

When Table 1 is examined, it is seen that the majority of the students made mistakes in the multiplication and division operations before the RME activities, and that the mistakes of the students in the multiplication and division operations decreased after the implementation of the activities. It can be said that the RME activities developed for the multiplication and division operations are effective in eliminating or reducing the mistakes that occur in the operations.

Discussion, Conclusion and Recommendations

Before the RME application, it was revealed that two primary school fourth grade students did not make any mistakes in the multiplication operations, one student made one mistake, three students made two mistakes, one student made three mistakes and three students made four mistakes. When the students' level of mistakes in the multiplication operations was examined, it was determined that three students made mistakes at the "low" level, three students made mistakes at the "medium" level and four students made mistakes at the "high" level. Failure to eliminate the mistakes of the students who make mistakes in the multiplication operation will result in unhealthy learning in advanced mathematics subjects. The mistakes made by the students regarding the multiplication operation stem from not knowing the meanings of the concepts related to multiplication, not understanding the problem situation related to multiplication, and operational mistakes. Not knowing the concepts such as "times" involved in multiplication operations causes students to make mistakes in multiplication. Knowing these concepts semantically will reduce the mistakes that students will make in multiplication operations because it has been stated that there is a relationship between the development of conceptual understanding and the development of operational skills in children (Canobi, 2005; Rasmussen, Ho & Bisanz, 2003). Ünlü and Ertekin (2012) stated that the reason for not being able to establish a relationship between concepts and operational skills is that the teaching process is based on memorization. It is also known that the concepts of multiplication are difficult for students to learn (Tanujaya, Prahmana & Mumu, 2017). This is a situation that prevents conceptual learning for multiplication. It has been revealed that students made mistakes in multiplication operations as a result of not understanding the problems that require multiplication. In order to eliminate this problem of understanding, the problems should be read more carefully, while the process of converting the verbal expression into numerical form should be operated. Since this result indicates the transformation of problems chosen from daily life into mathematical expressions, this problem can be eliminated by implementing the activities created on the basis of RME. This is evidence supporting that the implementation of RME-based activities can be effective in eliminating mistakes. It can be said that another factor leading students to make mistakes in multiplication operations is the inability to transfer them to the problem because they have not acquired conceptual knowledge. In this connection, Baroody (1999) stated that not having gained conceptual understanding causes learning without establishing meanings between concepts and problem situations. The low level of affective status of students towards mathematics enables them to read the problems quickly and solve them without making sense of them. Since this encourages students to have the desire to finish the questions immediately, it causes students to immediately engage in solving problems without paying much attention. As a result of carelessness, mistakes occur in the selection and construction of the operation for multiplication. The operational mistakes that occur in the multiplication operation arise from reasons such as multiplication with “0” and “1”, shifting the digits, forgetting to add the carry, rhythmic counting, and addition operations within multiplication operations. These results are similar to the results reported in the studies by Harris (2001), Barmby et al. (2009) and Anghileri (1989). Elimination of operational mistakes depends on students’ being more attentive, having adequate background knowledge about rhythmic counting, and fully internalizing the conceptual structure of multiplication.

During the implementation of the RME activities for the multiplication operations, the selection of the activities that the students encounter and perform in their close environment, in their lives, increased the students’ interest and participation in the lesson. Since the students realized that they would use multiplication everywhere as a result of their engagement in the activities, they willingly participated in all the work done during the lesson. It is stated that RME is suitable for use in educational environments since it allows students to take active role in lessons (Marsigit, Dhoruri & Mahmudi, 2007; Zaranis, 2016). Since the student takes an active role in the process, it has been determined that the conceptual and operational mistakes related to multiplication have decreased and disappeared. When the mistakes made by the students in the multiplication operations after the implementation of the RME activities were examined, it was revealed that eight students did not make any mistakes, and two students made one mistake each. When we look at the mistake levels of the students regarding the multiplication operation, it is seen that all the students are at the “low” level. Choosing and applying activities that appeal to students are of great importance in reducing mistakes. RME reduced students’ mistakes regarding the multiplication operations, and fostered a positive point of view in students towards mathematics. The RME activities improved students’ self-confidence towards multiplication. In general, it can be thought that mathematical activities related to multiplication prepared by being influenced by the experiential environment prevent the student from making mistakes about multiplication.

Before the RME application, it was revealed that two primary school fourth grade students made five mistakes, three students made six mistakes, and five students made seven mistakes regarding the division operations. When the levels of the mistakes made by the students in the division operations were examined, it was determined that ten students were at the “high” level. The reason why the students made such a high level of mistakes in the division operations may be that the operational process was unusual, complex and included too many conceptual structures. It has been revealed that the mistakes made by the students regarding the division operations stemmed from reasons such as the concepts of dividend, divisor, quotient and remainder, the use of “0” in division, not being able to continue the operation, and not understanding the problem situation regarding division. Students mostly have difficulty in acquiring the concepts related to the division operation (Simon, 1993). Students may make mistakes as a result of not learning the concepts in the division operation or confusing the semantic values of the concepts. Since there is a relationship between conceptual knowledge and operational skills (Robinson, Ninowsky, & Grey, 2006), the absence of conceptual learning also poses an obstacle to the development of operational skills. This is supported by the statement of Yorulmaz, Uysal, and Sidekli (2021) that conceptual learning is important in order to thoroughly learn the division operation. It can be said that mistakes are made because the concepts of dividend, divisor, quotient and remainder involved in the division operation are abstract and not sufficiently concretized. If the concepts are transformed into concrete objects, the operations related to the concepts will be fully performed as it will be easier to occupy space in the mind of the student. It can be said that the use of concrete objects in the teaching of concepts has an important place in eliminating mistakes and ensuring correct learning. Students make the mistake regarding the use of the number "0" while performing the division operation. Not adding the number “0” to the quotient section is caused by the student’s carelessness or lack of knowledge. Another reason for making the mistake is not knowing the following rule; when dividing, the number is taken down from the top after the subtraction, and if there is no divisor in the newly created dividend number, “0” is added to the quotient. Another mistake made in the division operation is the result of not continuing the operation as a result of not being able to find the divisor within the dividend by using the multiplication operation or by rhythmic counting. In problems that require division, since the problem is not understood by the student, it cannot be transformed into a mathematical expression. Since the process of transforming the problem situation into a mathematical expression is expressed as mathematization, it can be said that RME is the most effective approach to make this possible. For this reason, the activities used in the application process were prepared on the basis of RME.

During the implementation of the RME activities for the division operation, selection of the activities that the students would encounter and perform in their close environment, in their lives, increased the interest and participation of the students in the lesson and thus they became more ready to learn. The activities which are selected from environments to which students are familiar and in which they can put themselves in place of the characters are important to increase the participation of students with low classroom participation otherwise (Fauzan, Slettenhaar & Plomp, 2002; Barnes, 2005). Students who take an active part in the lesson and participate effectively will be able to better learn the use of the concepts of division and to perform division according to the rules. Students will be able to solve problems more easily if problem situations are created that students will find exciting, entertaining, pleasing and relevant to their needs in their daily lives. When the mistakes made by the students participating in the study after the RME application were examined, it was revealed that five students did not make any mistakes, four students made one mistake, and one student made two mistakes. When the level of the mistakes made by the students in the division operations was examined, it was seen that all the students are at the “low” level. The application of activities that motivate and encourage students to the mathematics lesson has an important place in reducing the mistakes related to the division operation. It can be said that RME-based activities reduce students’ mistakes about division and increase students’ interest in mathematics. After the RME activities, it is seen that the students are not afraid of performing division operations and it is easier to make sense of it.

When the students’ mistakes in the multiplication and division operations before and after the RME application were compared, it was found that there was a decrease in the number of mistakes. The decrease seen in the mistakes made for the multiplication and division operations shows that the RME-based activities are effective in eliminating mistakes. There are many studies that show that RME-based activities are effective on students (Aytekin Uskun, 2020; Barnes, 2005; Eade & Dickinson, 2006; Hansa, 2017; Kalaw, 2012; Marija, Lidija & Simona, 2000; Papadakis, Kalogiannakis & Zaranis, 2017; Peters, 2016, Topçu, 2021, Webb, Van Der Kooji & Geist, 2011; Yorulmaz, 2018). It has been revealed that problem situations created by establishing connections with daily life in line with RME are effective in eliminating mistakes. Makonye (2014) stated that the contextual problems in RME provide a better understanding of mathematical concepts. Again, Arsoetar and Sugiman (2019) stated that problems that are related to daily life are effective in the construction of mathematical knowledge. It can be stated that RME applications are an approach that should be preferred in eliminating the operational, conceptual and problem-related mistakes made by students. It can be said that eliminating the mistakes made by students in multiplication and division operations will increase the academic achievement in mathematics. In light of the results of the study, the following suggestions can be made.

- When multiplication and division operations are concretized using real-life problem situations, mistakes committed by students can be reduced or eliminated.
- Activities prepared in the context of GME for multiplication and division can be designed using digital content.
- Teachers can work individually with students on the concepts of multiplication and division in order to eliminate mistakes.
- RME can be used as an approach in the education processes of students who have learning difficulties in mathematics.
- RME can be used to correct the mistakes made by students in other subjects of mathematics.

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